Long Term Behaviour of Solutions of Nonlinear Parabolic Equations

by

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Abstract

This thesis is concerned with the long term behaviour of solutions of semilinear parabolic initial boundary value problems on bounded domains as well as linear parabolic equations with nonlinear boundary conditions.

We obtain several conditions on the nonlinear terms under which solutions converge to zero for all initial values. The rates at which this happens are examined. When the nonlinear term obeys suitable conditions the rate of decay is exponential. This rate of decay is governed by the first eigenvalue of a corresponding elliptic eigenvalue problem. This is done for a variety of types of boundary conditions. In each case a sharp estimate of the rate of decay is obtained.

All of the above concerns the situation where the zero solution to the corresponding elliptic steady state problem is a global attractor. We show that if we impose only the weaker condition that the first eigenvalue is positive, then the zero solution remains a local attractor. Conversely, if the first eigenvalue is negative, then the zero solution is unstable.

It is natural then to consider the case where the first eigenvalue is actually zero. Under suitable restrictions, we show that in this case the rate of decay is slower than exponential and obtain a sharp estimate for this rate.

Next we examine the case where the nonlinearity depends explicitly on time and obtain similar results. We also deal with the case where the domain is all of $\mathbb{R}^n$. In all of the above we are concerned with uniform convergence.

In the case where the equation is self-adjoint, the natural topology of convergence is in H^2. Again we obtain results concerning exponential and slower than exponential convergence. It transpires that convergence in H^2 also implies uniform convergence.

If the nonlinearity represents a strong enough source solutions may become unbounded. We obtain conditions under which the solutions grow exponentially. We also consider the situation where solutions blow up in finite time.

We then turn to the case where there are non-trivial steady states. We give sufficient conditions to ensure the existence of non-trivial steady states and examine their stability.

Finally we examine the situation where the equation itself is linear, but the boundary condition is not. Again we derive a sharp estimate of the rate of decay of solutions to zero. Using the methods of Leray-Schauder degree we consider the special case of the Steklov problem. We prove some results about the number of steady states as well as their stability.