NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SUPPLEMENTARY EXAMINATION

SMA1101 CALCULUS/ANALYSIS I

3 hours
March 2003

Answer **All** questions in **Section A** and **ANY FOUR** questions in **Section B.**

SECTION A

1. a) Evaluate

$$\int_0^{\pi} (x \sin x)^2 \, dx$$

b) $\int_0^{\ln 2} \sinh^4 x dx$

[3,3]

2. If $I_n = \int x^n e^{-x} dx$, show that $I_N = -x^n e^{-x} + nI_{n-1}$ and hence evaluate I_3.

[3,2]

3. (a) Prove that $\cosh 2x = \cosh^2 x + \sinh^2 x$

[2]

(b) Express in terms $\tanh^{-1} x$ in terms of logarithmic function, and hence, or otherwise, show that

$$\tanh^{-1} \alpha + \tanh^{-1} \beta = \tanh^{-1}\left(\frac{\alpha + \beta}{1 + \alpha\beta}\right)$$ [4]

4. Sketch and hence find the area enclosed by the curve $r = 3(1 - \sin\theta)$. [3,3]

5. Sketch the curve whose equation is $y = 9 - x^2$. Show that the tangent to the curve $y = 9 - x^2$ at $A(1,8)$ cuts the x-axis at $B(5,0)$. Find the area of the finite region bounded by AB, the x-axis and the curve $y = 9 - x^2$.

[3,4,4]

6. Find the arc length of the following curve $x = \frac{1}{3}(2t+7)^{3/2}$, $y = \frac{t^2}{2} + 3t$ from $t = 1$ to $t = 9$.

[6]

SECTION B: Answer any FOUR QUESTIONS in this section.

7. Show that $(z + z^{-1})^4 = z^4 + 4z^2 + 6 + 4z^{-2} + z^{-4}$ and that

$\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$.

Hence find

(a) $\int \cos^4\theta \; d\theta$

(b) $\int \sin^4\theta \; d\theta$

[3,6,6]

8. (a) Find the total length of the cardiod $r = a(1+\cos\theta)$.

[5]

(b) A curve has polar equation $r = ae^{2\theta}$, where a is a positive constant. At the points A and B on the curve the values of θ are 0 and $\pi/4$ respectively. Find in terms of a, θ and π:

(i) the area of the finite region bounded by the lines $\theta = 0$, $\theta = \pi/4$ and arc AB.

[5]

(ii) the length of arc AB.

[5]

(a) Differentiate $\sec^{n-2} x \tan x$ with respect to x. Hence or otherwise, show that if $n > 1$ and $I_n = \int \sec^n x \; dx$, then

$(n-1)I_n = \sec^{n-2} x + \tan x + (n-2)I_{n-2}$

Prove that $8\int_0^{\pi/4} \sec^5 x \; dx = 7\sqrt{2} + 3\log(\sqrt{2}+1)$

[4,6]

(b) Prove that $sech^{-1}x = \log\frac{\left(1+\sqrt{1-x^2}\right)}{x}$ [5]

10. a) Define a point of inflection.

The curve $y = x^3 + bx^2 + cx + d$ ($b, c,$ and d are constants) will have a point of inflection when $x = 1$ if b has which of the following values?

i)	2	ii)	-2
iii)	3	iv)	-3

Justify. [4]

b) If $y = e^{a\sin^{-1}x}$ show that

i) $\left(1-x^2\right)y_2 - xy_1 - a^2y = 0$

ii) $\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} - \left(n^2+a^2\right)y_n = 0$

where $y_n = \frac{d^n y}{dx^n}$. [4]

11. Use the Mean Value Theorem to prove the following identities:

a. i) $e^x - 1 < e$ for $0 < x < 1$

ii) $|\ln x - \ln y| < \frac{|x-y|}{2}$ for $x > y > 2$ [5]

b. Sketch the curve represented by

$$y = \frac{x^2 - x - 2}{x(x-3)(x-4)}$$

showing all the asymptotes and stationary points. [5]

c. Evaluate the following limits

i) $\lim_{x\to 3^+} \frac{x^2-9}{|x-3|}$ ii) $\lim_{x\to a} \frac{a-\sqrt{a^2-x^2}}{x^2}$ [5]

END OF QUESTION PAPER