NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1101

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

CALCULUS I:SUPPLEMENTARY

JULY 2004

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. If $f(x) = \dfrac{3x + |x|}{7x - 5|x|}$ evaluate (where possible);

(a) $\lim_{x \to 0^+} f(x)$

(b) $\lim_{x \to 0^-} f(x)$

(c) $\lim_{x \to 0} f(x)$. [3,3,3]

A2. Show that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$. [6]

A3. Solve the following inequality $\dfrac{2}{x-1} < \dfrac{3}{2x+1}$. [5]

A4. Find the derivative of $f(x) = \cos x$ from first principles. [5]

A5. (a) Find the exact value of the modulus, r and the argument, θ of $(1+i)^{22}$.

(b) Express $\dfrac{(1+i)^{11}}{(1-i)^{11}}$ in the form $x + iy$. [3,2]

A6. Use Leibniz Rule to find $\frac{d^6y}{dx^6}$ when $y = x^3 \cos x$. [4]

A7. Evaluate the following integrals:

(a) $\int (x+2)\sin(x^2+4x-6)dx$

(b) $\int \frac{\cot(\ln x)}{x}dx$

(c) $\int \frac{\sinh x}{\sqrt{\cosh x}}dx$. [2,2,2]

SECTION B: Answer THREE questions in this section [60].

B8. (a) Given the function $f(x) = \begin{cases} \sqrt{1-x^2}, & 0 \le x < 1 \\ 2, & x = 1 \\ 1, & 1 < x < 2 \end{cases}$

Find each of the following limits, if it exists:

(i) $\lim_{x \to 1^+} f(x)$

(ii) $\lim_{x \to 1^-} f(x)$

(iii) $\lim_{x \to 1} f(x)$. [3,3,3]

(b) Suppose $f'(x) = 2$ and that $f(0) = 5$ use the Mean Value Theorem to show that $f(x) = 2x + 5$ at every value of x. [6]

(c) Apply De Moivre's theorem to evaluate the integral

$$\int_0^{2\pi} e^{3x}\cos 5x dx$$

[5]

B9. (a) Use integration by parts to evaluate $\int e^{-2x}\cos 3x dx$

(b) Given that $t = \tan\frac{x}{2}$, show that $\sin x = \frac{2t}{1+t^2}$.

(c) Hence or otherwise evaluate $\int \frac{dx}{2+2\cos x - \sin x}$.

(d) The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the $y-axis$ to generate a solid. Find the volume of the solid. [5,4,6,5]

B10. (a) Suppose that $z = cos\theta + isin\theta$ show that $(z+z^{-1}) = z^4+4z^2+6+4z^{-2}+z^{-4}$

(b) Hence deduce that $cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$

(c) Apply De Moivre's theorem to evaluate the integral

$$\int_0^{\frac{\pi}{2}} e^{3x}\cos 5x dx$$

(d) Solve the equation $z^4 - 7z^3 + 11z^2 + z + 34 = 0$ given that $z = 4 - i$ is one solution. [7,7,6]

B11. (a) Sketch the graph of $y = \dfrac{x-1}{x^2(x-2)}$. [6]

(b) Sketch the graph of $y = \left|\dfrac{x-1}{x^2(x-2)}\right|$. [2]

(c) Show that $\lim\limits_{x\to\infty}\left(1+\dfrac{a}{x}\right)^x = e^a$

(d) Given that $f(x) = 2x^2 - 5x + 6$, use δ,ϵ method to prove that $\lim\limits_{x\to 2} f(x) = 4$

(e) Evaluate $\lim\limits_{x\to\infty}\left(\dfrac{2x-3}{3x-7}\right)^4$. [4,4,4]

END OF QUESTION PAPER