NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1101

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

CALCULUS I

NOV/DEC 2004

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. (a) Solve the following inequalities

(i) $|3x - 4| < 2$,

(ii) $\dfrac{x-1}{2x+3} \geq 1$.

(b) Suppose f and g are given by $f(x) = 6 - 3x$ and $g(x) = \dfrac{3}{(7-2x)}$.

Find

(i) $(gof)(x)$,

(ii) $g^{-1}(x)$,

(iii) $g(g^{-1}(g(x)))$. [5,4]

A2. Given that $f(x) = 2x^2 - 5x + 6$ use the $\epsilon - \delta$ definition of a limit to prove that $\lim_{x \to 2} f(x) = 4$. [4]

A3. Find the 10^{th} derivative of $x^3 \ln(3x + 1)$. [5]

B8. (a) Solve the equation $z^4 - 3z^3 + 4z^2 + 33z - 65 = 0$ given that $z = 2 - 3i$ is one solution. [4]

(b) Apply De Moivre's theorem to evaluate the integral

$$\int_0^{\frac{\pi}{2}} e^{2x} \sin 3x dx.$$

[5]

(c) Suppose $z = \cos\theta + i\sin\theta$.

(i) Show that $z + z^{-1} = 2\cos\theta$ and $z - z^{-1} = 2i\sin\theta$.

(ii) Show that $z^n + z^{-n} = 2\cos n\theta$ and $z^n - z^{-n} = 2i\sin\theta, \quad n \in N$.

(iii) Show that

$$(z + z^{-1})^6 = z^6 + 6z^4 + 15z^2 + 20 + 15z^{-2} + 6z^{-4} + z^{-6}$$

and deduce that

$$\cos^6\theta = \frac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10).$$

Hence evaluate $\int \cos^6\theta d\theta$. [11]

B9. (a) Sketch the graph of the function $f(x) = \dfrac{x+1}{(x-2)(3x+5)}$. [7]

(b) Show that the function $f(x) = \begin{cases} \sin \pi x, & x \leq 1 \\ x^3 - 1, & x > 1 \end{cases}$ is continous but not differentiable at $x = 1$. [4]

(c) Given that $f(x) = \cos x$. Find $f'(x)$ from first principles. [4]

(d) Suppose $f'(x) = \dfrac{1}{1+x^3}$ and $y = f(x^{\frac{1}{3}})$. Find $\dfrac{dy}{dx}$. Given that $f(1) = 2$ find the equation of the tangent $y = f(x^{\frac{1}{3}})$ when $x = 1$. [5]

B10. (a) Given that $t = \tan\dfrac{x}{2}$ show that $\sin x = \dfrac{2t}{1+t^2}$. [4]

(b) Hence or otherwise evaluate $\displaystyle\int \frac{dx}{2 + 2\cos x - \sin x}$ [6]

(c) The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the $y-$ axis to genarate a solid. Find the volume of the solid. [5]

(d) If $C_n = \int_0^{\frac{\pi}{2}} \cos^n x dx$ show that C_n satisfies the reduction formula

$$C_n = \frac{(n-1)}{n} C_{n-2}, \qquad n \geq 2.$$

[5]

END OF QUESTION PAPER