NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1101

## FACULTY OF APPLIED SCIENCES

### DEPARTMENT OF APPLIED MATHEMATICS

CALCULUS I: SUPPLEMENTARY EXAM

JULY 2005

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

**SECTION A: Answer ALL questions in this section [40].**

**A1.** (a) Solve the following inequalities

(i) $\dfrac{x+2}{x-3} \leq 2$. [3]

(ii) $|5-2x| \geq 3$. [4]

(b) (i) Define the composition $fog$ of the functions $f$ and $g$ and state when is this composition possible. [2]

(ii) If $(gof)(x) = |\sin x|$ and $(fog) = (\sin\sqrt{x})^2$, find $f(x)$ and $g(x)$. [5]

**A2.** Evaluate the following limits:

(a) $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-1}{x^2+2x-3}$. [3]

(b) $\lim\limits_{x\to\infty} \left(\sqrt{x+1}-\sqrt{x-1}\right)$. [3]

**A3.** Define the derivative of $f(x)$. Using first principles, show that the derivative of $f(x) = \sqrt{x+1}$ is $f'(x) = \dfrac{1}{2\sqrt{x+1}}$. [5]

**A4.** State Leibniz's theorem, and hence determine $f^5(x)$ if $f(x) = x^3 \cos 2x$. [4]

**A5.** Show that $\coth^{-1} x = \frac{1}{2} \ln \frac{x+1}{x-1}$, $\quad |x| > 1$.

Hence, find $\frac{d}{dx}[\coth^{-1}(x)]$. [6]

**A6.** Evaluate the following integral:

$$\int \frac{\sinh x}{\sqrt{\cosh 2x}} dx$$

[5]

**SECTION B: Answer THREE questions in this section [60].**

**B7.** (a) State Rolle's Theorem. [2]

(b) State and prove the Mean Value Theorem. [7]

(c) Hence, prove that $\frac{b-a}{1+b^2} < \arctan b - \arctan a < \frac{b-a}{1+a^2}$ if $a < b$. [7]

(d) Hence, show that $\frac{\pi}{4} + \frac{3}{25} < \arctan \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$. [4]

**B8.** (a) Given that $t = \tan \frac{x}{2}$ show that $\sin x = \frac{2t}{1+t^2}$.

(b) Hence or otherwise evaluate $\int \frac{dx}{2 + 2\cos x - \sin x}$.

(c) The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the $y - axis$ to generate a solid. Find the volume of the solid.

(d) If $C_n = \int_0^{\frac{\pi}{2}} \cos^n x dx$, show that $C_n$ satisfies the reduction formula

$$C_n = \frac{n-1}{n} C_{n-2}, \qquad n \geq 2$$

[4,6,5,5]

**B9.** (a) Express $5\theta$ in terms of $\cos\theta$ and hence solve the equation

$$16x^5 - 20x^3 + 5x - 1 = 0$$

(b) Apply de Moivre's theorem to evaluate the integral

$$\int_0^{\frac{\pi}{2}} e^{3x}\cos 5x dx$$

(c) Solve the equation $z^4 - 7z^3 + 11z^2 + z + 34 = 0$ given that $z = 4 - i$ is one solution. [7,7,6]

**B10.** (a) Given that $f(x) = 6x^2 + x - 12$, find the minimum value of $f(x)$ and the values of $x$ for which $f(x) = 0$. [5]

(b) Using the same axis, sketch the graphs of $y_1 = f(x)$ and $y_2 = \dfrac{1}{f(x)}$, labelling each clearly. [10]

(c) Deduce that there are four values of $x$ for which $[f(x)]^2 = 1$. Find these values correct to two decimal places. [5]

END OF QUESTION PAPER