NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1101

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

CALCULUS I

DECEMBER 2005

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. (a) Solve the following inequlities:

(i) $\dfrac{(x-1)^2}{x-1} < 1.$ [2]

(ii) $\sqrt{x} - 3 \leq \dfrac{2}{\sqrt{x}-2}.$ [4]

(b) Solve $4|x| = |x-1|$. [3]

A2. Evaluate

(a) $\displaystyle\lim_{x \to 8} \frac{\sqrt{9+2x}-5}{x^{\frac{1}{3}}-2}.$ [4]

(b) $\displaystyle\lim_{x \to \infty} \left(\sqrt{(x+2)(x+3)} - 8\right).$ [4]

A3. Find the derivative of $f(x) = \sin 3x$ from first principles. [5]

A4. Find the following integrals:

(a) $\int 2^{-x} \tanh 2^{1-x} dx.$ [3]

(b) $\int \frac{x}{\sqrt{x^2+x+1}} dx.$ [4]

A5. If $f(x) = \sqrt{x}$, $g(x) = \frac{x}{4}$ and $h(x) = 4x - 8$, find:

(a) $g(h(f(x)))$. [2]

(b) $f(h^{-1}(g(x)))$. [3]

A6. Show the first three non-vanishing terms of the Maclaurin series of $e^{-x}\cos(x^2)$. [6]

SECTION B: Answer THREE questions in this section [60].

B7. (a) A rectangular block has a base which measures x cm by $2x$ cm. Given that its volume is 243 cm^3, prove that the total surface area S cm^2 is given by:

$$S = 4x^2 + \frac{729}{x}.$$

Calculate the value of x for which the surface area is least. [7]

(b) A ladder 26m long rests on a horizontal ground and leans against a vertical wall. The foot of the ladder is pulled away from the wall at the rate of $4m/sec$. How fast is the top sliding down the wall when the foot is 10m from the wall. [7]

(c) The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y–axis to generate a solid. Find the volume of the solid. [6]

B8. (a) Sketch the graph of $y = \dfrac{2x^2 + x}{x + 1}$. [9]

(b) Show that the fuction

$$f(x) = \begin{cases} \sin \pi x, & x \leq 1 \\ x^3 - 1, & x > 1 \end{cases}$$

is continous but not differentiable at $x = 1$. [6]

(c) For the limit, $\lim_{x \to \frac{9}{4}} \dfrac{1}{x - 2} = 4$, find a δ that works for $\epsilon = 0.5$. [5]

B9. (a) Find the real and imaginary parts of $\dfrac{e^{(a+ib)x}}{a + ib}$, where $a, b \in \Re$. [7]

(b) Find the modulus and the argument of each root of the equation $z^2 + 4z + 8 = 0$. If the roots are denoted by α and β, simplify the expression:

$$\frac{\alpha + \beta + 4i}{\alpha\beta + 8i}.$$

[7]

(c) Apply De Moivre's theorem to evaluate the integral

$$\int e^{-2x} \sin 5x dx.$$

[6]

B10. (a) State and prove the Mean Value Theorem. [7]

(b) Use the mean value theorem to show that:

$$\sqrt{2} - 1 < \frac{\pi}{4}.$$

[**HINT** : Let $f(x) = \cos x, \quad 0 < x < \frac{\pi}{4}$]. [7]

(c) It took 20 seconds for a thermometer to rise from -10^{0}C to 100^{0}C when it was taken from a freezer and placed in boiling water. Show that somewhere along the way the mercury was rising at exactly 5.5^{0}C/sec. [6]

END OF QUESTION PAPER