NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA1101: CALCULUS

Exam

JANUARY 2008
Time : 3 hours

Candidates should attempt **ALL** questions from section A and **ANY THREE** questions from Section B.

SECTION A

A1. (a) Is it possible for the statement $\lim_{x\to 1} f(x) = 4$ to be true, and yet $f(1) = 2$? Explain your answer. [2]

(b) Evaluate the following limits:

(i) $\lim_{x\to 3^+} \dfrac{x^2-9}{|x-3|}$ [3]

(ii) $\lim_{x\to -\infty} xe^x$. [3]

(iii) $\lim_{x\to\infty} \left(\dfrac{2x-3}{3x-7}\right)^4$. [3]

A2. (a) Solve the inequality

$$\sqrt{x} - 3 \le \frac{2}{\sqrt{x}-2}$$

[4]

(b) Solve $4|x| = |x-1|$. [3]

A3. Show that the function

$$f(x) = \begin{cases} \sin \pi x, & x \leq 1 \\ \text{x}^3 - 1, & x > 1 \end{cases}$$

is continuous but not differentiable at $x = 1$. [3]

A4. Given $f(x) = \dfrac{3}{5x^2}$, find $f'(x)$ from first principles. [6]

A5. State Lebniz's rule,and hence find $y^{(6)}$ if $y(x) = e^{2x} \sin x$. [5]

A6. Prove that

$$\lim_{x \to \infty} \int_0^{2\pi} \frac{\sin n\pi}{x^2 + 1} dx = 0$$

[5]

A7. Prove that

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{4}$$

[3]

SECTION B

B8. (a) Given that $t = \tan \dfrac{x}{2}$, show that $\sin x = \dfrac{2t}{1 + t^2}$. [4]

(b) Hence or otherwise evaluate

$$\int \frac{dx}{2 + 2\cos x - \sin x}$$

[6]

(c) The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the $y - axis$ to generate a solid. Find the volume of the solid. [5]

(d) If $C_m = \displaystyle\int_0^{\frac{\pi}{2}} \cos^m x dx$, use integration by parts to show that C_m satisfies the reduction formula

$$C_m = \frac{m-1}{m} C_{m-2}, \quad m \geq 2$$

[5]

B9. (a) Find the real and imaginary parts of $\dfrac{e^{(a+ib)x}}{a+ib}$, where $a, b \in \Re$. [7]

(b) Find the modulus and the argument of each root of the equation $z^2 + 4z + 8 = 0$. If the roots are denoted by α and β, simplify the expression:

$$\frac{\alpha + \beta + 4i}{\alpha\beta + 8i}.$$

[7]

(c) Apply DeMoivre's theorem to evaluate the integral

$$\int e^{-2x} \sin 5x dx.$$

[6]

B10. (a) State Rolle's Theorem. [2]

(b) State and prove the Mean Value Theorem. [7]

(c) Hence, prove that $\dfrac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \dfrac{b-a}{1+a^2}$ if $a < b$. [7]

(d) Hence, show that $\dfrac{\pi}{4} + \dfrac{3}{25} < \tan^{-1} \dfrac{4}{3} < \dfrac{\pi}{4} + \dfrac{1}{6}$. [4]

B11. (a) State when a function is one to one. [2]

(b) If $f(x) = x^2 - 10x + 16, 2 < x < 8$

(i) Find $f(-1)$ and $f(3)$. [1,1]

(ii) Find the range of $f(x)$. [4]

(iii) After suitably restricting the domain of $f(x)$, find $f^{-1}(x)$ and state its domain. [5]

(iv) If $g(x) = \sqrt{x^2 - 8x + 16}$ find $(g \circ f)(x)$. [3]

(c) Prove that $\displaystyle\lim_{x \to 3} \frac{9 - x^2}{x - 3} = 6$ [4]

END OF QUESTION PAPER