

DEPARTMENT OF APPLIED MATHEMATICS

BSC HONOURS IN APPLIED MATHEMATICS: PART I

LINEAR ALGEBRA.

SUPPLEMENTARY EXAMINATION AUGUST 2004

Time : 3 hours

Answer any five questions. All questions carry equal marks

Q1. (a) Given that

$$R = \begin{pmatrix} 2 & 3 \\ -4 & 2 \\ 1 & 4 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 4 \\ -2 & 7 \end{pmatrix}, \quad T = \begin{pmatrix} 3x - y & 29 \\ -8 & z - 1 \\ -7 & 4x + y \end{pmatrix},$$

find the values of  $x, y$  and  $z$  if  $RS = C$ . [5]

(b) Prove that if  $A = [a_{ij}]$  is of order  $m \times n$  and if  $B = [b_{ij}]$  and  $C = [c_{ij}]$  are of order  $n \times p$ , then  $A(B + C) = AB + AC$ . [6]

(c) Given that

$$P = \begin{pmatrix} 4 & 0 & 4 \\ -1 & 0 & 1 \\ 1 & -3 & 0 \end{pmatrix},$$

find

(i)  $\text{Adj}(P)$ , [6]

(ii)  $P^{-1}$ . [3]

Q2. (a) The following forces act on a particle at  $P$  whose position vector is  $3i + j - 2k$ :  
 $F_1 = 2i - 3j + 5k$ ,  $F_2 = 3i - 2j - k$ .

(i) Find the magnitude of the resultant force of the forces acting on the particle. [2]

(ii) Write down the direction cosines of this resultant force. [1]

- (iii) Find the orthogonal projection of  $F_1$  on  $F_2$ . [4]  
 (iv) When  $F_2$  is applied to the particle alone, the object moves from  $P$  to a point  $Q$ , with position vector  $2i + 3k$ . Find the work done. [3]  
 (v) If the resultant force acts at the point  $A(2, 3, -1)$  find the moment of the resultant force about the point  $B(3, 2, 2)$ . [4]
- (b) Show that the lines given by the vector equations

$$\mathbf{r} = 10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$$

and

$$\mathbf{r} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k} + \mu(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$$

intersect and find the position vector of the point of intersection. [6]

- Q3. (a) The planes

$$x + 2y + z = 5 \quad \text{and} \quad 3x - y + 2z = 2$$

intersect on the line  $L$ .

- (i) Find the angle between the planes. [4]  
 (ii) Find the equation of  $L$  in  
 (a) parametric form,  
 (b) cartesian form. [5]
- (b) Find the vector equation of the plane which is perpendicular to the vector  $3\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$  which passes through the point with position vector  $2\mathbf{i} - \mathbf{j} - \mathbf{k}$ . [3]  
 (c) Find the perpendicular distance of the plane in 2(b) from the origin. [2]  
 (d) Prove that if  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$  form three edges of a parallelepiped all meeting at the same point then the volume of this figure is given by

$$|(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}|.$$

[3]

- (e) If  $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$ ,  $\mathbf{b} = \mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$ ,  $\mathbf{c} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ , find the volume of the parallelepiped formed by the vectors. [3]

- Q4. (a) Given that

$$A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 3 & 6 & 9 & 6 \\ 4 & 8 & 12 & k \end{pmatrix},$$

find the value(s) of  $k$  so that

- (i)  $R(A) = 1$   
 (ii)  $R(A) = 2$ .

[3]

(b) Suppose that  $M$  is invertible and say it is row reducible to the identity matrix  $I$  by the sequence of elementary operations  $e_1, \dots, e_n$ .

(i) Show that the sequence of elementary row operations to  $I$  yields  $M^{-1}$ .

(ii) Use this result to find the inverse of

$$M = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix},$$

[8]

(c) Solve the following system of equations using the LU decomposition method:

$$2x_1 + 6x_2 + 2x_3 = 4$$

$$-3x_1 - 8x_2 = 2$$

$$4x_1 + 9x_2 + 2x_3 = 5$$

[9]

**Q5.** (a) Write the vector  $\mathbf{V} = (1, -2, 5)$  as a linear combination of the vectors

$$(1, 1, 1), \quad (1, 2, 3), \quad (2, -1, 1).$$

[7]

(b) Find the conditions on  $a, b$  and  $c$  so that  $(a, b, c) \in \mathbf{R}^3$  belongs to the space generated by

$$u_1 = (2, 1, 0), \quad u_2 = (1, -1, 2), \quad u_3 = (0, 3, -4).$$

[7]

(c) Prove that if  $S = v_1, v_2, \dots, v_n$  is a basis for a vector space  $V$ , then every subset  $W$  with more than  $n$  elements is linearly dependent. [6]

**Q6.** (a) Solve the following system of equations using the Gauss-Jordan elimination method:

$$x + 2y - 3z = 3$$

$$2x - y - z = 11$$

$$3x + 2y + z = -5$$

[7]

(b) For the system

$$x + y + \alpha z = 0$$

$$x + y + \beta z = 0$$

$$\alpha x + \beta y + z = 0,$$

(i) show that it has non-trivial solution if and only if  $\alpha = \beta$ . [4]

(ii) find the non-trivial solutions, if  $\alpha = 2$ . [4]

(c) Write the matrix  $E = \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix}$  as a linear combination of the matrices

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix}.$$

[5]

Q7. (a) Let  $u, v$  and  $w$  be independent vectors. Show that  $u + v, u - v$  and  $u - 2v + w$  are also independent. [5]

(b) Find a basis for the space spanned by the sequence of vectors:

$$v_1 = (1, -2, 0, 0, -3), \quad v_2 = (2, -5, -3, -2, 6),$$

$$v_3 = (0, 5, 15, 10, 0), \quad v_4 = (2, 6, 18, 8, 6).$$

[5]

(c) Let  $V$  be the space generated by the polynomials

$$v_1 = t^3 - 2t^2 + 4t + 1$$

$$v_2 = 2t^3 - 3t^2 + 9t - 1$$

$$v_3 = t^3 + 6t - 5$$

$$v_4 = 2t^3 - 5t^2 + 7t + 5.$$

Find

(i) basis of  $V$

(ii) dimension of  $V$ . [6]

(d) Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{pmatrix}.$$

[4]

END OF QUESTION PAPER