NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1102

DEPARTMENT OF APPLIED MATHEMATICS

BSc HONOURS IN APPLIED MATHEMATICS: PART I

LINEAR ALGEBRA.

DECEMBER 2004

Time : 3 hours

Answer any **five** questions. All questions carry **equal** marks

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Q1. (a) Given that

$$R = \begin{pmatrix} 2 & 3 \\ -4 & 2 \\ 1 & 4 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 4 \\ -2 & 7 \end{pmatrix}, \quad T = \begin{pmatrix} 3x - y & 29 \\ -8 & z - 1 \\ -7 & 4x + y \end{pmatrix},$$

find the values of x, y and z if $RS = T$. [5]

(b) If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, show that $A^2 - 4A - 5I = 0$. [5]

(c) Given that

$$P = \begin{pmatrix} 4 & 0 & 4 \\ -1 & 0 & 1 \\ 1 & -3 & 0 \end{pmatrix},$$

find

(i) $\text{Adj}(P)$, [7]

(ii) P^{-1}. [3]

Q2. (a) The following forces act on a particle at P whose position vector is $3i + j - 2k$.:
$F_1 = 2i - 3j + 5k, \qquad F_2 = 3i - 2j - k.$

(i) Find the magnitude of the resultant force of the forces acting on the particle. [2]

(ii) Write down the direction cosines of this resultant force. [1]

(iii) Find the orthogonal projection of F_1 on F_2. [4]

(iv) When F_2 is applied to the particle alone, the object moves from P to a point, Q, with position vector $2i + 3k$. Find the work done. [3]

(v) If the resultant force acts at the point $A(2, 3, -1)$ find the moment of the resultant force about the point $B(3, 2, 2)$. [4]

(b) Show that the lines given by the vector equations

$$\mathbf{r} = \mathbf{10i} + \mathbf{2j} + \mathbf{3k} + \lambda(\mathbf{2i} - \mathbf{3j} + \mathbf{k})$$

and

$$\mathbf{r} = \mathbf{3i} - \mathbf{j} - \mathbf{2k} + \mu(\mathbf{i} + \mathbf{3j} + \mathbf{k})$$

intersect and find the position vector of the point of intersection. [6]

Q3. (a) Find two planes whose intersection is the line

$$x = 3 + 2\mu; \quad y = -4 + 7\mu; \quad z = 1 + 3\mu,$$

where $-\infty < \mu < +\infty$. [5]

(b) Show that the planes $x + 2y - 2z = 3$ and $2x + 4y - 4z = 7$ are parallel. Hence find the distance between these planes. [5]

(c) Find the cartesian equation of the plane which is perpendicular to the vector $\mathbf{3i} - \mathbf{6j} - \mathbf{2k}$ which passes through the point with position vector $\mathbf{2i} - \mathbf{j} - \mathbf{k}$. [3]

(d) Prove that if **a**, **b**, **c** form three edges of a parallelepiped all meeting at the same point then the volume of this figure is given by

$$|(a \times b).c|.$$

[3]

(e) If $\mathbf{a} = \mathbf{3i} + \mathbf{2j} + \mathbf{5k}, \mathbf{b} = \mathbf{i} - \mathbf{3j} - \mathbf{2k}, \mathbf{c} = \mathbf{2i} - \mathbf{j} + \mathbf{4k}$, find the volume of the parallelepiped formed by the vectors. [3]

Q4. (a) Write the vector $\mathbf{V} = (\mathbf{1}, -\mathbf{2}, \mathbf{5})$ as a linear combination of the vectors

$$(\mathbf{1}, \mathbf{1}, \mathbf{1}), \quad (\mathbf{1}, \mathbf{2}, \mathbf{3}), \quad (\mathbf{2}, -\mathbf{1}, \mathbf{1}).$$

[7]

(b) Find the conditions on a, b and c so that $(a, b, c) \in \mathbf{R}^3$ belongs to the space generated by

$$\mathbf{u_1} = (\mathbf{2}, \mathbf{1}, \mathbf{0}), \quad \mathbf{u_2} = (\mathbf{1}, -\mathbf{1}, \mathbf{2}), \quad \mathbf{u_3} = (\mathbf{0}, \mathbf{3}, -\mathbf{4}).$$

[7]

(c) Prove that if $S = v_1, v_2, \dots .v_n$ is a basis for a vector space V, then every set W with more than n elements is linearly dependent. [6]

Q5. (a) Solve the system

$$\begin{aligned} 3y + 2x &= z + 1 \\ 3x + 2z &= 8 - 5y \\ 3z - 1 &= x - 2y \end{aligned}$$

using Cramer's rule. [7]

(b) For the system

$$\begin{aligned} x + 5y + \alpha z &= 0 \\ -2x + 6y + 2z &= 0 \\ \alpha x - y + z &= 0, \end{aligned}$$

(i) find the values of α for which the system has non-trivial solutions. [4]

(ii) find the non-trivial solutions, if $\alpha = 3$. [4]

(c) Write the matrix $E = \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix}$ as a linear combination of the matrices

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix}.$$

[5]

Q6. (a) Let $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ be independent vectors. Show that $\mathbf{u} + \mathbf{v}, \mathbf{u} - \mathbf{v}$ and $\mathbf{u} - 2\mathbf{v} + \mathbf{w}$ are also independent. [5]

(b) Find the dimension and a basis for the solution space $\mathbf{V}$ of the system

$$\begin{aligned} x_1 + 2x_2 + 2x_3 - x_4 + 3x_5 &= 0 \\ x_1 + 2x_2 + 3x_3 + x_4 + x_5 &= 0 \\ 3x_1 + 6x_2 + 8x_3 + x_4 + 5x_5 &= 0 \end{aligned}$$

[5]

(c) Find the rank of the matrix

$$M = \begin{pmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{pmatrix}.$$

[4]

Q7. (a) It is given that

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 3 & 6 & a & 12 \end{pmatrix}$$

(i) Reduce A to row echelon form. [4]

(ii) Hence find the determinant of the matrix. [2]

(iii) For what real value of a is the matrix singular? [2]

(iv) Hence, or otherwise, solve the following system of linear equations with $a = 10$:

$$\begin{aligned} x + y + z + w &= 3 \\ x + 2y + 3z + 4w &= -3 \\ x + 3y + 6z + 10w &= 3 \\ 3x + 6y + 10z + 12w &= -3. \end{aligned}$$

[6]

(b) Prove that if $A = [a_{ij}]$ is of order $m \times n$, $B = [b_{ij}]$ and $C = [c_{ij}]$ are of order $n \times p$, then $A(B + C) = AB + AC$. [6]

END OF QUESTION PAPER