NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1102

DEPARTMENT OF APPLIED MATHEMATICS

BSc Honours in Applied Mathematics: Part I

Linear Algebra.

SUPPLEMENTARY EXAMINATION JULY 2005

Time : 3 hours

Answer any **five** questions. All questions carry **equal** marks

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Q1. (a) Given that

$$A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 3 & 6 & 9 & 6 \\ 4 & 8 & 12 & k \end{pmatrix},$$

find the value(s) of k so that

(i) $R(A) = 1$

(ii) $R(A) = 2$

(iii) $R(A) = 3$.

[7]

(b) Prove that if $A = [a_{ij}]$ is of order $m \times n$ and if $B = [b_{ij}]$ is of order $n \times p$, then $(AB)^t = B^t A^t$ [5]

(c) Solve the following system of equations using the LU decomposition:

$$\begin{aligned} x + 2y + 3z &= 5 \\ 2x + 5y + 3z &= 3 \\ x + 8z &= 17 \end{aligned}$$

[8]

Q2. (a) The following forces act on a particle at P whose position vector is $3i + j - 2k$.:
$F_1 = 2i - 3j + 5k, \qquad F_2 = 3i - 2j - k.$

(i) Find the magnitude of the resultant force of the forces acting on the particle. [2]

(ii) Write down the direction cosines of this resultant force. [1]

(iii) Find the orthogonal projection of F_1 on F_2. [4]

(iv) When F_2 is applied to the particle alone, the object moves from P to a point, Q, with position vector $2i + 3k$. Find the work done. [3]

(v) If the resultant force acts at the point $A(2, 3, -1)$ find the moment of the resultant force about the point $B(3, 2, 2)$. [4]

(b) Show that the lines given by the vector equations

$$\mathbf{r} = 10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - 3\mathbf{j} + \mathbf{k})$$

and

$$\mathbf{r} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k} + \mu(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$$

intersect and find the position vector of the point of intersection. [6]

Q3. (a) The planes

$$x + 2y + z = 5 \quad \text{and} \quad 3x - y + 2z = 2$$

intersect on the line L.

(i) Find the angle between the planes. [4]

(ii) Find the equation of L in

(a) parametric form,

(b) cartesian form.

[5]

(b) Find the vector equation of the plane which is perpendicular to the vector $3\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$ which passes through the point with position vector $2\mathbf{i} - \mathbf{j} - \mathbf{k}$. [3]

(c) Find the perpendicular distance of the plane in 2(b) from the origin. [2]

(d) Prove that if $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ form three edges of a parallelepiped all meeting at the same point then the volume of this figure is given by

$$|(a \times b).c|.$$

[3]

(e) If $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}, \mathbf{b} = \mathbf{i} - 3\mathbf{j} - 2\mathbf{k}, \mathbf{c} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$, find the volume of the parallelepiped formed by the vectors. [3]

Q4. (a) Given that the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 6 & 4 \\ 4 & 8 & \lambda \end{pmatrix}.$$

is singular, find the value(s) of λ. [3]

(b) Suppose that M is invertible and say it is row reducible to the identity matrix I by the sequence of elementary operations $e_1, ..., e_n$.

(i) Show that the sequence of elementary row operations to I yields M^{-1}.

(ii) Use this result to find the inverse of

$$M = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix},$$

[8]

(c) Solve the following system of equations using the LU decomposition method:

$$\begin{aligned} 2x_1 + 6x_2 + 2x_3 &= 4 \\ -3x_1 - 8x_2 &= 2 \\ 4x_1 + 9x_2 + 2x_3 &= 5 \end{aligned}$$

[9]

Q5. (a) Write the vector $\mathbf{V} = (1, -2, 5)$ as a linear combination of the vectors

$$(1,1,1), \quad (1,2,3), \quad (2,-1,1).$$

[7]

(b) Find the conditions on a, b and c so that $(a, b, c) \in \mathbf{R}^3$ belongs to the space generated by

$$u_1 = (2,1,0), \quad u_2 = (1,-1,2), \quad u_3 = (0,3,-4).$$

[7]

(c) Prove that if $S = v_1, v_2, ...v_n$ is a basis for a vector space V, then every subset W with more than n elements is linearly dependent. [6]

Q6. (a) Solve the following system of equations using the Gauss-Jordan elimination method:

$$\begin{aligned} x + 2y - 3z &= 3 \\ 2x - y - z &= 11 \\ 3x + 2y + z &= -5 \end{aligned}$$

[7]

(b) For the system

$$\begin{aligned} x + y + \alpha z &= 0 \\ x + y + \beta z &= 0 \\ \alpha x + \beta y + z &= 0, \end{aligned}$$

(i) show that it has non-trivial solution if and only if $\alpha = \beta$. [4]

(ii) find the non-trivial solutions, if $\alpha = 2$. [4]

(c) Write the matrix $E = \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix}$ as a linear combination of the matrices

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix}.$$

[5]

Q7. (a) Let u, v and w be independent vectors. Show that $u+v, u-v$ and $u-2v+w$ are also independent. [5]

(b) Find a basis for the space spanned by the sequence of vectors:

$$v_1 = (1, -2, 0, 0, -3), \quad v_2 = (2, -5, -3, -2, 6),$$
$$v_3 = (0, 5, 15, 10, 0), \quad v_4 = (2, 6, 18, 8, 6).$$

[5]

(c) Let V be the space generated by the polynomials

$$\begin{aligned} v_1 &= t^3 - 2t^2 + 4t + 1 \\ v_2 &= 2t^3 - 3t^2 + 9t - 1 \\ v_3 &= t^3 + 6t - 5 \\ v_4 &= 2t^3 - 5t^2 + 7t + 5. \end{aligned}$$

Find

(i) basis of V

(ii) dimension of V.

[6]

(d) Find the rank of the matrix

$$A = \begin{pmatrix} 1 & -3 & 1 & 2 \\ 1 & -2 & 3 & -1 \\ 2 & -3 & -4 & 1 \\ 3 & 0 & -15 & -3 \end{pmatrix}.$$

[4]

END OF QUESTION PAPER