NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA1103 DISCRETE MATHEMATICS

SUPPLEMENTARY EXAMINATION

January 2003
Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY FOUR** questions from Section B.

SECTION A

A1 Let $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

Prove, by induction, that

$$\begin{pmatrix} 1 & 3^k - 1 \\ 0 & 3^k \end{pmatrix}$$

for $k = 1,2,3,\ldots\ldots$

[6]

A2. (i) Prove that the linear function $f(x) = ax + b$ where a,b are constants and $a \neq 0$ has an inverse function, and determine $f^{-1}(x)$.

(ii) Does a constant function have an inverse? Justify your answer.

[4,2]

A3. Let X be a nonempty set and $P(X)$ its power set. Define a relation on P by

$$A \sim B \text{ if and only if } A \subseteq B.$$

Investigate whether $\sim$ is reflective, symmetric, antisymmetric or transitive.

[6]

A4. Consider the following sets:

$A = \{n \in N$: n is divisible by 2$\}$

$B = \{m \in N$: m is the sum of two odd numbers$\}$

Show that $A = B$.

[7]

A5. Use proof by contradiction to show that $\sqrt{2}$ is an irrational number.

[6]

A6. Let $\oplus$ be a binary operation on Z defined by

$$\forall m, n \in Z : m \oplus n = m + mn$$

(a) Discuss whether $\oplus$ is
(i) commutative
(ii) associative

[2,2]

(b) (i) Does Z have an identity with respect to $\oplus$?
(ii) Which elements of Z are idempotent with respect to $\oplus$?

[3,2]

SECTION B

B7. a) Prove that for any sets A, B if

$$A \subseteq B \text{ then } P(A) \subseteq P(B).$$

b) Give a counter example to show that $P(A) \cup P(B) = P(A \cup B)$ is **NOT** true in general.

c) Let A, B, X, Y be sets such that $B = X \cup Y$.
Show that

$$A \times B = (A \times X) \cup (A \times Y)$$

[15]

B8. (a) Consider the function

$$f : x \rightarrow x^2 + 1 \;,\; \boldsymbol{R} \rightarrow \boldsymbol{R}$$

State, with reasons, whether or not f is

i) injective (i.e. one to one);
ii) surjective (i.e. onto)

(b) Describe a function g formed by restricting the domain and codomain of f that is bijective, and write down the inverse of g.

[15]

B9. (a) Give the truth table for the following formula:

$$((p \land \neg q) \lor r) \rightarrow ((r \lor p) \leftrightarrow (r \lor q)$$

(b) Let P and Q be statements and T a tautology. Give the truth table for each of the following compound statements:

i) $\text{T} \rightarrow (P \land Q)$
ii) $(P \lor \neg \text{T}) \Leftrightarrow \neg Q$

[15]

B10. a) Let R be a relation in $N \times N$ which is defined by

$$(a,b)R(c,d) \text{ iff } ad = bc$$

Show that R is an equivalence relation.

b) Let R be an equivalence relation on a set A. Prove that

i) For any $a \in A$, $a \in [a]$.

ii) For any $a, b \in A$, $aRb \Leftrightarrow [a] = [b]$.

iii) For any $a, b \in A$, if $\neg(aRb)$ then $[a] \cap [b] = \Phi$.

iv) Any two equivalence classes are either equal or disjoint.

[15]

B11. (a) Let G be the set of nonzero complex numbers and let H be the subset consisting of these complex numbers with unit modulus. Assuming G is a group with respect to multiplication of complex numbers, show that H is a subgroup of G.

(b) Let G be the set of pairs (a,b) of real numbers with $a \neq 0$. Define multiplication on G by

$$(a,b) \cdot (c,d) = (ac, bc + d).$$

Prove that G is a group.

Let H be the subset consisting of pairs of the form $(1,b)$. Show that H is a subgroup of G.

[15]

END OF QUESTION PAPER.