NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1103

DEPARTMENT OF APPLIED MATHEMATICS

SMA1103 DISCRETE MATHEMATICS

DECEMBER 2004

Time : 3 hours

This paper contains TWO sections. Answer ALL questions in section A and ANY FOUR questions from section B.

SECTION A: Answer ALL questions in this section [40].

A1. Let A and B be sets.

(a) Prove that $A \times B = \phi$ if and only if $A = \phi$ or $B = \phi$.

(b) Give a counter example to show that

$$P(A) \cup P(B) = P(A \cup B)$$

is not true in general. [$P(A)$ is the power set of A]

[3,2]

A2. $A = \{n \in \aleph : n \text{ is divisible by } 2\}$; $B = \{m \in \aleph : m \text{ is the sum of two odd numbers}\}$. Show that $A = B$. [6]

A3. Let

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}.$$

Prove, by induction, that

$$A^k = \begin{pmatrix} 1 & 3^k - 1 \\ 0 & 3^k \end{pmatrix}$$

for $k = 1, 2, 3, - - - -$. [6]

A4. Use induction or otherwise to prove that

$$1^2 + 2^2 + 3^2 + - - - + n^2 = \frac{1}{6}n(n+1)(2n+1)^2.$$

[6]

A5. Let $\oplus$ be a binary operation on $\mathbf{Z}$ defined by

$$m \oplus n = m + mn, \in \mathbf{Z}.$$

(a) Discuss whether $\oplus$ is

(i) commutative

(ii) associative.

(b) (i) Does $\mathbf{Z}$ have an identity element with respect to $\oplus$?

(ii) Which elements of $\mathbf{Z}$ are idempotent with respect to $\oplus$?

[8]

A6. (a) Let X be a set. When is $R \subseteq X \times X$ said to be an equivalent relation?

(b) Determine whether or not the following relations from $\mathbf{Z}$ to $\mathbf{Z}$ are equivalence relations:

(i) mRn iff $m + n = 0$.

(ii) mQn iff $m - n$ is even.

[3;3;3]

SECTION B: Answer TWO questions in this section [60].

B7. (a) Let G be the set of non-zero complex numbers and let H be the subset constisting of those complex numbers with unit modulus. Assuming G is a group with respect to multiplication of complex numbers, show that H is a sub group of G.

(b) Let G be the set of pairs(a, b) of real numbers with $a \neq 0$. Define multiplication on G by

$$(a, b).(c, d) = (ac, bc + d).$$

Prove that G is a group.

(c) Let H be the subset constisting of pairs of the form $(1, b)$. Show that H is a subgroup of G.

[15]

B8. (a) Consider the function

$$f :\rightarrow x^2 + 1.\Re \rightarrow \Re.$$

State with reasons, whether or not f is

(i) injective.

(ii) surjective.

(b) Describe a function g formed by restricting the domain and codomain of f that is bijective, and write down the inverse of g.

[15]

B9. Write down the truth tables for

(a)

$$(A \rightarrow B) \wedge (A \rightarrow \neg A).$$

(b)

$$(\neg A \vee B) \vee \neg C.$$

(c)

$$((A \wedge \neg B) \vee C) \rightarrow ((A \vee C) \leftrightarrow (B \vee C)).$$

[3,5,7]

B10. Use the Principle of Mathematical induction to prove that

(a)

$$1^3 + 2^3 + 3^3 + - - - - + k^3 = \frac{1}{4}k^2(k + 1)^2.$$

(b)

$$\sqrt{n} < 1 + \frac{1}{\sqrt{2}} + - - - - - + \frac{1}{\sqrt{n}} < 2\sqrt{n}, \quad n \geq 2,$$

hence, deduce that the series

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

is divergent.

[5,10]

B11. (a) Prove that for any sets A, B if $A \subseteq B$ then $P(A) \subseteq P(B)$.

(b) Let A, B, X and Y be sets such that $B = X \cup Y$. Show that

$$A \times B = (A \times X) \cup (A \times Y).$$

(c) Let X be a nonempty set and $P(X)$ its power set. Define a relation $\sim$ on $P(X)$ by

$$A \sim B \quad \text{iff} \quad A \subseteq B.$$

Investigate whether $\sim$ is reflexive, symmetric, anisymmetric or transitive.

[15]

END OF QUESTION PAPER