NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA1103

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 1103:DISCRETE MATHEMATICS

DECEMBER 2011: EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY THREE** questions from **Section B** (20 marks each).

SECTION A

Answer ALL questions[40]

A1. Are the following statements about sets (i) always true, (ii) sometime true and (iii) never true:

(a) If A is finite, A is bounded [1]

(b) If A is infinite, A is bounded. [1]

(c) If A is a subset of [-23,79], A is finite. [1]

(d) If A is a subset of [-23,79], A is unbounded. [1]

A2. Prove the following:

(a) $(-x)(-y) = xy$. [3]

(b) $(x^{-1})^{-1} = x$. [5]

A3. While on a Saturday shopping spree Jennifer and Tiffany witnessed two men driving away from a jewelery shop, just before a burglar alarm started to sound. Although everything happened rather quickly, when the two young ladies were questioned they were able to provide the following information about the license plate (which consisted of three letters followed by four digits) on the get-away car. Tiffany was sure that the second letter on the plate was either O or Q and the last digit was either a 3 or 8. Jeniffer told the investigator that the first letter was either a C or G and that the first digit was definitely a 7. How many different license plates will the police have to check out. [5]

A4. Use mathematical induction to prove the following:

$$\alpha + \alpha e + \alpha e^2 + ... + \alpha e^n = \frac{\alpha(e^{n+1} - 1)}{e - 1}$$

. [5]

A5. Show that

$$(((A \cup B) \cap C)^c \cup B^c)^c = B \cap C.$$

[6]

A6. Let s, t and u denote the following primitive statements:
s: Phyllis goes out for a walk.
t: The moon is out.
u: It is snowing.
Write an english statement represented by the following symbolic expression.
$(t \wedge \neg u) \rightarrow s$. [6]

A7. Determine if the following statement about natural numbers is an order or an equivalence relation, 'x times y is a square of a number'. [6]

SECTION B

Answer ANY THREE questions[60]

B8. (a) Prove that between any two real number there is always a real number. [3]

(b) If
$x \equiv 2(mod3)$,
$x \equiv 3(mod5)$,
$x \equiv 6(mod7)$
find the value of x and express it in the form $x \equiv y(modm)$ [4]

(c) Prove $(A\Delta B)^c = A\Delta B^c$, where A and B are non-empty. [6]

(d) Prove that if $k \equiv 1(mod3)$ then $k^3 \equiv 1(mod9)$. [5]

(e) Find x and y if $(x-1, 4) = (2y+1, 2x+y)$. [2]

B9. (a) Define the logical terms

(i) tautology, [2]

(ii) contradiction. [2]

(b) Prove that the conditional operation distributes over the operation of conjunction: $p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r)$. [5]

(c) Find the exact rational numbers represented by the decimals $0.5\overline{231}$ [3]

(d) Let $f : A \to B$, $g : B \to C$ and $h : C \to D$ be bijections. Show that

(i) $h \circ (g \circ f) : A \to D$. [3]

(ii) $(h \circ g \circ f)^{-1} = f^{-1} \circ g^{-1} \circ h^{-1}$. [5]

B10. (a) Prove that

$$\sum_{k=1}^{n}(2k-1) = n^2.$$

[3]

(b) At the Haddon and Sly cooperation Mrs Forster operates the Quick Snack Coffee Shop. The menu at her shop is limited: six kinds of muffins, eight kinds of sandwiches and five beverages (hot coffee, hot tea, iced tea, cola and orange juice). Ms Dodd, an editor at Chronicle, sends her assistant Carl to get her lunch-either a muffin and hot beverage or a sandwich and a cold beverage.

(i) In how many ways can Carl purchase Ms Dodd's lunch? [4]

(ii) Explain the rules you used (i). [4]

(c) Consider the functions f: A $\to$ B and g:B $\to$ C Prove the following:

(i) If f and g are one-to-one then, the composite $(g \circ f)$ is one-to-one. [3]

(ii) if f and g are onto then, the composite $(g \circ f)$ is onto. [3]

(d) Write $(A \cup B \cup C)$ as a disjoint union of sets. [3]

B11. (a) Show that

(i) $\sqrt{2} + \sqrt{3}$ is irrational. [5]

(ii) $3^{3n+3} - 26n - 27$ is a multiple of 169 for all natural numbers n. [5]

(b) Find the closed form for the sequence $u_0 = 0$, $u_1 = \dfrac{5}{2}$ and $u_{n+2} = \dfrac{3}{2}u_{n+1} + u_n$. [5]

(c) Palindromes are numbers such that the integers are read the same whether from right to left or left to right, such as 101, 3223, 74547 and so on. Find the sum of all palindromes between 1000 and 10000. [5]

END OF QUESTION PAPER