NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1103

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 1103:DISCRETE MATHEMATICS

AUGUST 2012: SUPPLEMENTARY EXAMINATION

Time : 3 hours

Candidates should attempt **ALL** questions from **Section A** (40 marks) and **ANY THREE** questions from **Section B** (20 marks each). GOOD LUCK!

SECTION A

Answer ALL question [40]

A1. Use the principle of mathematical induction to prove the following:

(a) $2^n \geq 2n$ [5]

(b) If $A = \begin{pmatrix} 1 & a \\ 0 & 2 \end{pmatrix}$ then $A^n = \begin{pmatrix} 1 & (2^n - 1)a \\ 0 & 2^n \end{pmatrix}$ [5]

A2. Prove De-Morgan's law $(A \cup B)^c = A^c \cap B^c$. [5]

A3. Define the binary operation Δ for all subsets of A and B by $A\Delta B = (A \cup B) - (A \cap B)$.

(a) Show that Δ is commutative. [5]

(b) Show that $A\Delta B = (A - B) \cup (B - A)$. [5]

A4. Prove that for all positive integers n, if n^2 is even, then n is even. [5]

A5. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Show that if:

(a) $g \circ f$ is surjective, so is g. [5]

(b) $g \circ f$ is injective, so is f. [5]

SECTION B

Answer ANY three questions[60]

B6. (a) Prove that between any two real number there is always a real number. [3]

(b) If
$x \equiv 2(mod3)$
$x \equiv 3(mod5)$
$x \equiv 6(mod7)$
Find the value of x and express it in the form $x \equiv y(modm)$ [4]

(c) Prove the following identity for non-empty sets A,and B
$(A\Delta B)^c = A\Delta B^c$ [6]

(d) Margie, Mimi, Rachel and April ran a race. Asked how they made out, they replied:

- Margie: "April won and Mimi was second."
- Mimi; "April was second and Rachel third."
- April:"Rachel was last and Margie was second."

If each of the girls made one and only one true statement, who won the race? [5]

(e) Find x and y if $(x - 1, 4) = (2y + 1, 2x + y)$. [2]

B7. (a) Find the exact rational numbers represented by the decimals $0.5\overline{231}$. [3]

(b) Let $f : A \rightarrow B$, $g : B \rightarrow C$ and $h : C \rightarrow D$ be bijections. Show that

(i) $h \circ (g \circ f) : A \rightarrow D$. [3]

(ii) $(h \circ g \circ f)^{-1} = f^{-1} \circ g^{-1} \circ h^{-1}$. [5]

(c) Use induction to prove $\cos\theta(\cos 2\theta) \circ \circ \circ (\cos 2^n\theta) = \dfrac{\sin(2^{n+1}\theta)}{2^{n+1}\sin\theta}$. [4]

(d) Prove that the conditional operation distributes over the operation of conjuction:
$p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$. [5]

B8. (a) Prove that

$$\sum_{k=1}^{n}(2k - 1) = n^2.$$

[3]

(b) At the Haddon and Sly cooperation Mrs Forster operates the Quick Snack Coffee Shop. The menu at her shop is limted: six kinds of muffins, eight kinds of sandwiches and five beverages (hot coffee, hot tea, iced tea, cola and orange juice). Ms Dodd, an editor at Chronicle, sends her assistant Carl to get her lunch-either a muffin and hot beverage or a sandwich and a cold beverage.

(i) In how many ways can Carl purchase Ms Dodd's lunch? [4]

(ii) Explain the rules you used (i). [4]

(c) Consider the functions $f : A \to B$ and $g : B \to C$. Prove the following:

(i) if f and g are ono-to-one, then the composite $(g \circ f)$ is one-to-one. [3]

(ii) if f and g are onto, then the composite $(g \circ f)$ is onto. [3]

(d) Write $(A \cup B \cup C)$ as a disjoint union of sets. [3]

B9. (a) Prove that if R is an equivalence relation then, R^{-} the inverse ralation for R is also an equivalence ralation. [5]

(b) Find the closed form for the sequence $u_0 = 3$, $u_1 = 8$ and $u_{n+2} = 12u_{n+1} - 20u_n$. [4]

(c) Find the value of n so that $\dfrac{n!}{(n-6)!} = 120\dfrac{n!}{(n-3)!}$. [4]

(d) Lynn and Patti decide to buy a PowerBall ticket. To win the grand prize they must match five numbers from 1 to 49 inclusively and also select an integer from 1 to 42 inclusively.

(i) In how may ways can they select the six numbers. [5]

(ii) Explain the rule you used in (i). [2]

END OF QUESTION PAPER