NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1103

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 1103:DISCRETE MATHEMATICS

OCTOBER 2012: TEST1

Time : 2 hours

Candidates should attempt **ALL** questions

SECTION A

A1. Prove the following for $x, y, z \in \mathbb{R}$

(a) $x * 0 = 0$. [here $*$ means multiplication] [5]

(b) $(-x)y = -(xy)$. [5]

A2. Prove the density of real numbers. [5]

A3. (a) Prove that $\sqrt{2} + \sqrt{3}$ is irrational. [5]

(b) State without the proof the completeness axiom. [3]

A4. Find the exact fraction for $4.5\overline{362}$. [4]

A5. Prove that if $a|b$ and $a|c$, then $a|(mb + nc)$ $\forall a, b, c, m, n \in \mathbb{N}$. [5]

A6. (a) Use Euclid's algoritm to find the GCD for (776,544). [5]

(b) Express the GCD in the form $GCD(776, 554) = \alpha 776 + \beta 554$, where $\alpha, \beta \in \mathbb{N}$. [5]

A7. Prove that if 5 divides $4x + 3$, then 5 divides $4x^2 - 5x - 6$. [5]

A8. Prove that $(A\Delta B)^c = A\Delta B^c$, where Δ is the symmetric difference. [6]

A9. Let $S(n) = \sum_{r=1}^{n} \frac{1}{r^2}$. Prove by mathematical induction that

$$\sum_{r=1}^{n} (2r + 1)S(r) = (n + 1)^2 S(n) - n.$$

[6]

A10. Prove that if $n = 2k$, $\forall k \in \mathbb{N}$, then $\frac{n!}{2^k}$ is an integer. [6]

Total marks: 70 Enjoy!!!!

END OF QUESTION PAPER