NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 1111: MATHEMATICS FOR SCIENCE 1

DECEMBER 2004

TIME: 3 HOURS

Candidates should attempt ALL questions from section A and ANY THREE questions from section B

Section A: Answer all questions in this section. [40]

A1 Suppose $A=\begin{bmatrix}1 & -1 & 1\\ 1 & 1 & 2\\ 2 & -1 & 4\end{bmatrix}$ and $B=\begin{bmatrix}6 & 3 & -3\\ 0 & 2 & -1\\ -3 & -1 & 2\end{bmatrix}$

Find AB and hence A^{-1}

[6 marks]

A2 Differentiate the following with respect to x.

(a) $y = x^x$ **[4 marks]**

(b) Calculate the stationary points (if they exist) of:

i) $y = x - In(1+x)$

ii) $y = 2x^3 + 3x^2 - 12x + 5$

[4 +4marks]

A3 Express $z = \dfrac{6-8i}{1+3i}$ in the form $a+bi$, and calculate $z\bar{z}$, where $\bar{z}$ is the conjugate of z.

[6 marks]

A4 Evaluate the following limits where possible

(a) $\lim\limits_{x\to\infty} \dfrac{2x^2-5x+2}{5x^2-7x-6}$

(b) $\lim\limits_{x\to\infty} \left(1-\dfrac{3}{x}\right)^{2x}$

(c) $\lim\limits_{x\to\infty} \dfrac{Inx}{e^x}$

[3+3+3marks]

A5 Find the integral of $I = \int \dfrac{1}{\sqrt{1-x^2}}\,dx$

[4marks]

A6 (a) A curve has the equation $x^2 + y^2 = 4$,

Show that $\dfrac{d^2y}{dx^2} = \dfrac{-4}{y^3}$

[3marks]

Section B: Answer THREE questions in this section. [60]

B7 (a) Find the eigenvalues and eigenvectors of;

$$\begin{bmatrix} 2 & 0 & 1 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

[10marks]

(b) Solve the following system of equations

(i)
$$\begin{aligned} 3x - y + 2z &= 5 \\ 2x + y - z &= 2 \\ 4x - 2y - 2z &= -1 \end{aligned}$$

(ii)
$$\begin{aligned} x - y - 4z &= 1 \\ 2x + 5y - z &= 2 \\ 3x + 2y - 3z &= -1 \end{aligned}$$

[5+5 marks]

B8 (a) Prove that $Sin7\theta = 7\cos^5\theta\sin\theta - 35\cos^4\theta\sin^3\theta + 21\cos^2\theta\sin^5\theta - \sin^7\theta$

[7marks]

Hence solve $7x^3 - 35x^2 + 21x - 1 = 0$

[7marks]

(b) Suppose $z = \cos\theta + i\sin\theta$ show that

(i) $z - \frac{1}{z} = 2i\sin\theta$ and

(ii) $z^n - \frac{1}{z^n} = 2i\sin n\theta, \quad n \in \mathrm{N}$

[6marks]

B9 (a) Find for the function $y = 4x^6 + x^5 - x^3$, $\frac{d^4y}{dx^4}$, hence or otherwise express the function as a Maclaurin series up to and including the term in x^4.

[6marks]

(b) Find an equation of the tangent and normal to the graph of $y = x^3 + 3x^2 - 4x + 1$ at the point where the value of the second derivative is zero.

[6marks]

(c) Find the stationary points and determine their nature if $y = x^3e^{-x}$.

[8marks]

B10 (a) Prove that $I_n = \int \sin^n x\, dx$ is $I_n = -\frac{1}{2}\cos x \sin^{n-1} x + \frac{n-1}{2} I_{n-2}$.

[10marks]

(b) Use Simpson's rule with 5 ordinates to estimate $\int_0^1 e^x dx$.

[5marks]

(b) Find the volume V contained when the curve $y = 4x+4$ is rotated about the x-axis through 360^0 between $x = 0$ and $x = 3$.

[5marks]

END OF QUESTION PAPER