NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1111

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA1111: MATHEMATICS FOR SCIENCE 1

DECEMBER 2005

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Sections B.

SECTION A: Answer ALL questions in this section [40].

A1. Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 5 & 8 & 9 \end{pmatrix}.$$

Find $\mathbf{A}^{-1}$. [6]

A2. Differentiate the following with respect to x,

(a) $y = x^{\cos x}$. [3]

(b) $y = \sin(\cos x)$. [3]

A3. Evaluate the following limits

(a) $\lim_{x\to\infty} \dfrac{1+x}{2+x}$, [3]

(b) $\lim_{x\to\infty} (\sqrt{x+1} - \sqrt{x})$, [5]

(c) $\lim_{x\to 0} \dfrac{1-\cos x}{x^2}$. [4]

A4. Given that $z_1 = 3 + 2i$ and $z_2 = -1 + i$, calculate

(a) $z_1 z_2$, [2]

(b) $\bar{z_1} z_1$ where $\bar{z_1}$ is the conjugate of z_1, [2]

(c) $\dfrac{z_1}{z_2}$. [2]

A5. Find the integral of $I = \int \sqrt{(a^2 - x^2)}dx$, where a is a constant and $|x| < a$. [5]

A6. Investigate continuity and differentiability of the function

$$f(x) = \begin{cases} \sin 3\pi x, & \text{if } x \leq 1 \\ x^3 - 1, & \text{if } x > 1 \end{cases}$$

at the point $x = 1$. [5]

SECTION B: Answer THREE questions in this section [60].

B7. (a) Solve the following system using Cramer's rule

$$\begin{aligned} 3y + 2x &= z + 1 \\ 3x + 2z &= 8 - 5y \\ 3z - 1 &= x - 2y. \end{aligned}$$

[5]

(b) For the following matrix, determine the eigenvalues and the associated eigenvectors

$$\begin{pmatrix} -2 & -3 & -3 \\ -1 & 0 & -1 \\ 5 & 5 & 6 \end{pmatrix}.$$

[10]

(c) Find all the roots of the equation

$$z^6 = -64.$$

[5]

B8. (a) Prove that $I_n = \int \tan^n \theta d\theta = \dfrac{\tan^{n-1}\theta}{n-1} - I_{n-2}$ for $n > 2$. [6]

(b) Use the Simpson's rule with five ordinates to evaluate the integral

$$I = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - \frac{1}{2}\sin^2\theta}}.$$

[7]

(c) Compute a solution for $2\sin x = x$ by Newton-Raphson method, starting from $x_0 = 2$. Perform three iterations. [7]

B9. (a) Suppose $z = \cos\theta + i\sin\theta$ show that

(i) $z^n + z^{-n} = 2\cos n\theta$. [3]

(ii) $z^n - z^{-n} = 2i\sin n\theta$. [3]

(b) Express $\cos 6\theta$ and $\sin 6\theta$ in terms of powers of $\cos\theta$ and $\sin\theta$. [7]

(c) Evaluate the following integral $\int e^{4x}\cos 5x dx$.

[7]

B10. (a) Expand the function $f(x) = xe^{4x}$ about $x = 0$ using Maclaurin's series. Obtain the first three nonzero terms. [7]

(b) Expand the function $f(x) = \cos x$ about the point $x = \frac{\pi}{3}$. Obtain the first three terms. [6]

(c) Find the critical points of $y = x^2e^{-x}$, and determine wether they are maxima or minima. [7]

END OF QUESTION PAPER