NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED METHEMATICS

JULY 2001 SUPPLEMENTARY EXAMINATION

SMA 1112 PREPARATORY MATHEMATICS

July 2001
3 hours

2 pages

Answer **ALL** questions in Section A and any **THREE** from Section B.

Section A. Answer all questions from this section [40 marks]

1. Find the maximum value of k for which the expression $x^2 + 2kx + 4$ is positive for all values of x.
[5 marks]

2. If z = $(1+i)/\sqrt{2}$, express z in the form z = r $(\cos\theta + i\sin\theta)$. Hence evaluate z^{20}.

[5 marks]

3. Express the following in partial fractions:

(a) $\dfrac{4x+21}{x^2+3x-4}$, (b) $\dfrac{x^4+2x+4}{(2x^2+3)(x-2)}$

[6 marks]

4. If **A** = $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$, calculate $\mathbf{A}^2$, $\mathbf{A}^T$, **A** – **I**, and $\mathbf{A}^{-1}$, if it exists.
(T denotes transpose and **I** is the unit matrix).

[10 marks]

5. Find the following limits:

(a) $\lim\limits_{x\to\infty}\dfrac{x^2-3x}{4x^2+5}$, (b) $\lim\limits_{x\to 0}\dfrac{\sin x}{x}$

[6 marks]

6. If y = $\left[1/\sqrt{1-x^2}\right]\sin^{-1}x$, deduce that $(1-x^2)\dfrac{dy}{dx} = 1 + xy$

[4 marks]

7. Evaluate $\int x\sin x dx$.

[4 marks]

Section B. Answer any THREE questions from this section [60 marks]

8. Given $\mathbf{A} = \begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & 5 \\ 4 & 1 & a^2 - 14 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 2 \\ a+2 \end{bmatrix}$,

   find the values of a such that $\mathbf{Ax} = \mathbf{b}$ has (a) a unique solution, (b) no solution and (c) an infinite number of solutions.

   [20 marks]

9. A town has two banks A and B and at the end of each year each bank retains 60% of its customers and the remaining 40% switch banks. Denote by x_n and y_n the number of customers of banks A and B respectively at year n. Write down the relationship between (x_n, y_n) and (x_{n+1}, y_{n+1}) in the form

   $$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_n \\ y_n \end{bmatrix}$$

   Find A and its eigenvalues and eigenvectors.

   If initially bank A has 60% of the customers, find the distribution of the customers after (a) one year, (b) five years and (c) an infinite number of years.

   [20 marks]

10. If $y = x^3 e^{-x}$, compute y' and y''. Use Leibnitz's rule to find $y^{(n)}$ for $n \geq 3$. Determine the two values of x which give stationary values of y. Are these values maximum, minimum or neither?

    Expand y in a Taylor series about the origin up to and including the term x^5.

    [20 marks]

10. If C is the curve $y = x^2$ between x = 0 and x = 2, sketch C. Find (a) the length of C and (b) the area between C and the x-axis.

    The curve C is rotated through 2π about the x-axis, calculate (a) the surface area and (b) the volume of the figure generated by C.

    [20 marks]

END