NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 1112: PREPARATORY MATHEMATICS

Supplementary Exam

TIME: 3 HOURS

Candidates should attempt ALL questions from section A and ANY THREE questions from section B

Section A: Answer all questions in this section. [40]

A1 If $\mathbf{A}=\begin{bmatrix}2 & 1\\ 4 & 2\end{bmatrix}$, calculate $\mathbf{A^2}$, $\mathbf{A^T}$, $\mathbf{A-I}$ and $\mathbf{A^{-1}}$, if it exists.
(**T** denotes transpose and **I** is the identity matrix)

[10marks]

A2 Express the following in partial fractions:

(a) $\dfrac{4x+21}{x^2+3x-4}$, (b) $\dfrac{x^4+2x+4}{(2x^2+3)(x-2)}$

[4 +4marks]

A3 Find the following limits:

(a) $\lim\limits_{x\to\infty}\dfrac{x^2-3x}{4x^2+5}$ (b) $\lim\limits_{x\to 0}\dfrac{\sin x}{x}$

[8 marks]

A4 Evaluate

(a) $\int x \sin x dx$ (b) $\int_0^3 \frac{2x+3}{2x+1} dx$ **[4+4 marks]**

A5 If $y = (1-x)^2 e^{2x}$ show that

$$(1-x)\,dy/dx + 2xy = 0$$

[6marks]

Section B: Answer THREE questions in this section. [60]

B6 (a) Use De Moivre's theorem to express $\sin 4\theta$ and $\cos 4\theta$ as polynomials in $\cos\theta$ and $\sin\theta$

(b) Suppose $z = \cos\theta + \sin\theta$ show that

(i) $z - \frac{1}{z} = 2\sin\theta$ and

(ii) $z^n - \frac{1}{z^n} = 2\sin\theta, n \in \mathrm{N}$

c) Express $z = \frac{6-8i}{1+3i}$ in the form $a+bi$, and calculate $z\bar{z}$, where $\bar{z}$ is the conjugate of z. **[20marks]**

B7 (a) Given that $x^4 + x^2y^3 - y^5 = 2x+1$

Use implicit differentiation to find $\frac{dy}{dx}$

(b) If $y = x^3e^{-x}$, find y' and y", hence find the power series of y up to and including the term in x^3.

(c) Find the stationary points and determine their nature if $y = x^3e^{-x}$.

[20marks]

B8 (a) Suppose $A=\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$. Find det A and A^{-1}

(b) Solve the following systems of equations

(i)
$$\begin{aligned} x+2y-z&=-5 \\ 2x-y+z&=-3 \\ x-y-3z&=-3 \end{aligned}$$

(ii)
$$\begin{aligned} x-y+z&=0 \\ x+2y-z&=0 \\ 2x+y+3z&=0 \end{aligned}$$

[20marks]

B9 Evaluate the following;

(a) $\int_5^7 \frac{2-x^2}{(x+3)(x^2-8x+16)}dx$

(b) $\int_{-\pi}^{\pi} \sin^2 3x dx$

(c) $\int x \ln x dx$

(d) Calculate the equation of the tangent and the normal to the curve $y=x^3+2x^2-4x-3$ at the point (-2;5). **[20marks]**

END OF QUESTION PAPER