

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 1112: PREPARATORY MATHEMATICS

DECEMBER 2004

TIME: 3 HOURS

Candidates should attempt ALL questions from section A and ANY THREE questions from section B

Section A: Answer all questions in this section. [40]

A1 Find the following limits:

(a) $\lim_{x \rightarrow 1} \frac{x-1}{x^2+x-2}$

(b) $\lim_{x \rightarrow \infty} x \sin \frac{1}{x}$

[8 marks]

A2 Express the following in partial fractions:

(a) $\frac{4x+21}{x^2+3x-4}$, and hence or otherwise evaluate $\int \frac{4x+21}{x^2+3x-4} dx$

[4 +4marks]

A3 If $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 2 \\ 2 & -1 & 4 \end{pmatrix}$, calculate A^2 , A^T , $A-I$ and A^{-1} , if it exists.

(T denotes transpose and I is the identity matrix)

[10marks]

- A4 Evaluate $\int \ln x \, dx$ (a) and $\int \sin^{-1} x \, dx$ (b) [4+4 marks]

A5 If $\sin y = y \cos 2x$ show that

$$\frac{dy}{dx} = \frac{-2y \sin 2x}{\cos y - \cos 2x}$$

[6marks]

Section B: Answer THREE questions in this section. [60]

- B6 (a) Given that $x^4 + x^2 y^3 - y^5 = 2x + 1$
Use implicit differentiation to find $\frac{dy}{dx}$
- (b) If $y = x^3 e^{-x}$, find y' and y'' , hence find the power series of y up to and including the term in x^3 .
- (c) Find the stationary points and determine their nature if $y = x^3 e^{-x}$.

[20marks]

- B7 (a) Use De Moivre's theorem to express $\sin 5\theta$ and $\cos 5\theta$ as polynomials in $\sin \theta$ and $\cos \theta$ respectively.

(b) Suppose $z = \cos \theta + i \sin \theta$ show that

(i) $z - \frac{1}{z} = 2i \sin \theta$ and

(ii) $z^n - \frac{1}{z^n} = 2i \sin n\theta, n \in \mathbb{N}$

- (c) Express $z = \frac{3-4i}{2+6i}$ in the form $a + bi$, and calculate $z \bar{z}$, where \bar{z} is the conjugate of z .

[20marks]

B8 (a) Suppose $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$. Find $\det A$ and A^{-1}

(b) Solve the following systems of equations using Gaussian Elimination Method.

$$3x - y - z = 2$$

$$x + y + z = 4$$

$$4x - y + z = 7$$

(c) Write the argument matrix of;

$$x + y + z = 7$$

$$x - y + 2z = 9$$

$$2x + y - z = -1$$

Hence or otherwise solve this system of equation.

[20marks]

B9 (a) Find the integral of the following;

(i) $\int \frac{1}{a^2 + x^2} dx$

(ii) $\int \frac{1}{5 - 4x - x^2} dx$

(b) Evaluate $\int_{-2}^2 (3x^2 - x + 1) dx$

(c) Find the volume obtained when the curve $y = 16 - x^4$ is rotated about the x -axis through 360° , between $x=0$ and $x=2$.

[20marks]

END OF QUESTION PAPER