

FACULTY OF APPLIED SCIENCES  
DEPARTMENT OF APPLIED MATHEMATICS  
SUPPLEMENTARY EXAM: PREPARATORY MATHEMATICS

JULY 2005

Time : 3 hours

Candidates should attempt ALL questions from Section A and ANY THREE questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. Find the following limits:

(a)  $\lim_{x \rightarrow 1} \frac{x-1}{x^2+x-2}$

(b)  $\lim_{x \rightarrow 0} \frac{x - x \cos x}{x - \sin x}$

[3+5]

A2. Express the following in partial fractions:

$\frac{4x+21}{x^2+3x-4}$ , and hence or otherwise evaluate  $\int \frac{4x+21}{x^2+3x-4} dx$

[4+4]

A3. If  $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 2 \\ 2 & -1 & 4 \end{pmatrix}$ , calculate  $A^2$ ,  $A^T$ ,  $A - I$  and  $A^{-1}$ , if it exists.

[10]

A4. Evaluate

(a)  $\int \ln x dx$ ,

(b)  $\int \sin^{-1} x dx$

[4+4]

- A5. If  $\sin y = y \cos 2x$  show that
- $$\frac{dy}{dx} = \frac{-2y \sin 2x}{\cos y - \cos 2x}$$

[6]

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SECTION B: Answer THREE questions in this section [60].

- B6. (a) Given that  $x^4 + x^2y^3 - y^5 = 2x + 1$ . Use implicit differentiation to find  $\frac{dy}{dx}$ .  
 (b) If  $y = x^3e^{-x}$ , find  $y'$  and  $y''$ , hence find the power series of  $y$  up to and including the term in  $x^3$ .  
 (c) Find the stationary points and determine their nature if  $y = x^3e^{-x}$ . [20]

- B7. (a) Use De Moivre's theorem to express  $\cos 4\theta$  and  $\sin 4\theta$  as polynomials in  $\cos \theta$  and  $\sin \theta$  respectively.  
 (b) Suppose  $z = \cos \theta + i \sin \theta$  show that  
 (i)  $z - \frac{1}{z} = 2i \sin \theta$  and  
 (ii)  $z^n - \frac{1}{z^n} = 2i \sin n\theta$   
 (c) Express  $z = \frac{3-4i}{2+6i}$  in the form  $a + bi$ , and calculate  $z\bar{z}$ , where  $\bar{z}$  is the conjugate of  $z$ . [20]

- B8. (a) Suppose  $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$ . Find  $\det A$  and  $A^{-1}$ .

- (b) Solve the following systems of equations using the Gaussian elimination method

$$\begin{aligned} 3x - y - z &= 2 \\ x + y + z &= 4 \\ 4x - y + z &= 7 \end{aligned}$$

- (c) Write the augmented matrix of

$$\begin{aligned} x + y + z &= 7 \\ x - y + 2z &= 9 \\ 2x + y - z &= -1 \end{aligned}$$

Hence or otherwise solve this system of equations. [20]

B9. (a) Find the integrals of the following:

(i)  $\int \frac{x^2}{x^2+1} dx$

(ii)  $\int \frac{1}{5-4x-x^2} dx$

(b) Evaluate  $\int_1^{-1} 6x^2 - 2x + 2 dx$

(c) Find the volume obtained when the curve  $y = 16 - x^4$  is rotated about the x-axis through one revolution, between  $x = 0$  and  $x = 2$ . [20]

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END OF QUESTION PAPER