NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1112

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA1112: PREPARATORY MATHEMATICS

DECEMBER 2005

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Sections B.

SECTION A: Answer ALL questions in this section [40].

A1. Solve the following equations, giving your answer correct to two significant figures

(a)

$$4^{2x+1} \cdot 5^{x-2} = 6^{1-x},$$

[5]

(b)

$$4^x - 2^{x+1} - 3 = 0.$$

[5]

A2. The expression

$$x^3 + px^2 + qx + 6$$

has the same remainder when divided by $x+1$ and $2-x$. Given that the remainder when the expression is divided by $x+3$ is -60, find the values of p and q. [5]

A3. Express $\sin 5\theta$ as a polynomial in $\sin\theta$. [5]

A4. Prove the following trignometric identity

$$\tan A + \cot A = \frac{2}{\sin 2A}.$$

Hence, or otherwise, find the values of θ lying between 0 and 2π which satisfy the equation

$$\tan(\theta + \frac{\pi}{4}) + \cot(\theta + \frac{\pi}{4}) = 4.$$

[3,4]

A5. (a) Find a term independent of x in the expression of

$$\left(x - \frac{2}{5x}\right)^6.$$

[3]

(b) Find all values of λ_0 for which the matrix

$$\begin{bmatrix} \lambda_0 - 3 & -1 \\ -6 & \lambda_0 + 2 \end{bmatrix}$$

is singular.

[5]

A6. Evaluate

$$\int x^2 e^{ax} dx \quad , \quad a - constant.$$

[5]

SECTION B: Answer THREE questions in this section [60].

B7. (a) Given that

$$Z = 32[\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}]$$

(i) show Z on an argand diagram, [3]

(ii) calculate all values of $Z^{\frac{1}{5}}$ and illustrate them on an argand diagram, [8]

(iii) express each complex number calculated in (ii) in the form $re^{i\theta}$. [5]

(b) Given that $Z_1 = r_1[\cos(\theta_1) + i\sin(\theta_1)]$ and $Z_2 = r_2[\cos(\theta_2) + i\sin(\theta_2)]$, derive

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)].$$

[4]

B8. (a) Let

$$A = \begin{bmatrix} -1 & 6 & 4 & 8 \\ 3 & 5 & 0 & 6 \\ 7 & 0 & 6 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$$

and let B be the matrix $B = A^2$. Find $2b_{43}A$. [7]

(b) Consider the system of linear equations

$$x + 2y + 3z = 0$$

$$3z + 2x + 5y = 1$$

$$8z + x = -1$$

where x, y, z are unknowns.

(i) Write down the coefficient matrix and the augmented matrix of the system of equations. [3]

(ii) Write down the system in matrix form i.e in the form $Ax = B$. [1]

(iii) Use the fact that the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

is

$$\begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

or otherwise, to solve the system of equations above. [4]

(c) Find the inverse function, $f^{-1}(x)$, of $f(x) = \frac{6-2x}{3}$. [5]

B9. (a) Evaluate the following limits

(i)

$$\lim_{x\to\infty} \left(\frac{2x-3}{3x+7}\right)^4,$$

[3]

(ii)

$$\lim_{x\to 0} \left(\frac{1 - 2\cos x + \cos 2x}{x^2}\right),$$

[4]

(iii)

$$\lim_{n\to\infty} \left(\frac{n+1}{n-2}\right)^{3n+2}.$$

[4]

(b) Given

$$x^2(x^2+y^2) = y^2,$$

show that

$$\frac{dy}{dx} = \frac{x(2x^2+y^2)}{y(1-x^2)}.$$

[4]

(c) The formula $s(t) = -16t^2 + 48t$ gives the height in metres of an object t seconds after it is thrown vertically at a speed of $48m/s$ from ground level. The formula is valid until the object returns to ground level. How high above ground level will the object reach?

[5]

B10. (a) Evaluate each of the following indefinite integrals

(i)

$$\int \frac{1}{x^2 - 3x + 2} dx.$$

[3]

(ii)

$$\int 3^{\sqrt{x}} dx.$$

[3]

(b) Find the area of the solid that is bounded by the following equations: $y = x^4$ and $y = 2 - x^2$.

[4]

(c) Using integration by parts, show that if

$$I_n = \int_0^{\pi} e^{-x} \sin^n x \, dx$$

then

$$(1+n^2)I_n = n(n-1)I_{n-2}.$$

Hence or otherwise evaluate

$$\int_0^{\pi} e^{-x} \sin^4 x \, dx.$$

[10]

END OF QUESTION PAPER