NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA1116

SUPPLEMENTARY EXAMINATION

DEPARTMENT OF APPLIED MATHEMATICS

SMA1116 ENGINEERING MATHEMATICS 1A

MARCH 2003

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY FOUR** questions from Sections B

SECTION A: Answer ALL questions in this section [45].

A1. (a) Find the domain of the function: $f(x) = \dfrac{x^2+3}{x^2-x+2}$

(b) Find the limit, if it exists, $\lim_{x\to\frac{\pi}{2}} \dfrac{\cos x}{1-\sin x}$ [4+4]

A2. (a) Consider a moving body B whose position at time t is given by:

$$R(t) = t^2\mathbf{i} + t^3\mathbf{j} + 2t\mathbf{k}$$

Find the velocity, the speed and acceleration of B when $t = 2$. [2+2+2]

(b) Find the equation of the plane with normal $\mathbf{N} = 3\mathbf{i}\text{-}4\mathbf{j}+5\mathbf{k}$ and containing the point $P(1, 1, -1)$. [2]

A3. (a) Use the substitution $u^2 = \cosh x - 1$ to find the integral: $\displaystyle\int \frac{\tanh x}{2\sqrt{\cosh x - 1}}dx$.

(b) Evaluate the definite integral: $\displaystyle\int_1^2 \frac{4x+6}{x^2+3x+1}dx$. [4+4]

A4. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ if $\sin y = y \cos 2x$. [4+4]

A5. Find the parametric equation of the line L :

(a) through the point $P(2,5,-3)$ and in the direction of $\mathbf{v} = 4\mathbf{i} - 5\mathbf{j} + 7\mathbf{k}$.

(b) through the point $P(1,2,-4)$ and $Q(3,-7,2)$.

[4+4]

SECTION B: Answer FOUR questions in this section [60].

B6. Find the critical values (maxima, minima and points of inflection) of the function: $f(x) = \frac{x^3}{x^2+3}$.

Sketch the graph of the function.

[10+5]

B7. (a) If $J_n = \int (1-x^3)^n dx$, where n is an integer, use the method of integration by parts to show that

$$(3n+1)J_n = x(1-x^3)^n + 3nJ_{n-1}.$$

Use the result to evaluate $\int_0^1 (1-x^3)^4 dx$.

[5+5]

(b) Evaluate the improper integral $\int_{-1}^{0} \frac{x^2+1}{x^2} dx$. [5]

B8. (a) In physics it is shown that when air resistance is ignored, the horizontal range R of a projectile is given by:

$$R = \frac{v_0^2}{g} \sin 2\theta, \quad 0 < \theta \leq \frac{\pi}{2},$$

where v_0^2 is the initial velocity, g is the acceleration of gravity, and θ is the angle of elevation or departure.

At what angle θ should the projectile be launched in order to achieve (*i*) minimum range? (*ii*) maximum range? [5+5]

(b) Find the area of the region bounded by the graphs of
$y = \sin x$ and $y = \cos x$ on $[\frac{\pi}{4}, \frac{5\pi}{4}]$. [5]

B9. (a) Sketch the region bounded by the graphs of $y = x + 2$, $y = x$, $x = 0$, and $x = 3$.

(b) Find the volume of a solid formed by revolving the region bounded by the graphs of $y = x + 2$, $y = x$, $x = 0$, and $x = 3$ about the x-axis.

[5+10]

B10. (a) Express $z = \dfrac{(1+i)^{13}}{(1-i)}$ in modulus and argument form.

(b) The points $z_1 = 6 + 8i$ and $z_2 = 4 - 3i$ are given in a complex plane. Find the set of points representing the bisector of the angle formed by vectors z_1 and z_2.

(c) Given that $z = -i$ is a root of the polynomial

$$P(z) \equiv z^5 - 2z^4 + 10z^3 - 20z^2 + 9z - 18,$$

find the values of the other four roots.

[4+4+7]

B11. The solid paraboloid is formed by revolving the region bounded by the graphs of $y = 4ax$, $x = 0$ and $x = a$.

(a) Show that the coordinates of the centre of mass are $(\frac{2a}{3}, 0)$.

(b) Find the moment of inertia about the x-axis of the solid of revolution.

[8+7]

END OF QUESTION PAPER