NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 1116: ENGINEERING MATHS 1A

July 2004: Supplementary Exam

TIME: 3 HOURS

Candidates should attempt ALL questions from section A and ANY THREE questions from section B

Section A: Answer all questions in this section. [40]

A1 Evaluate the following integral;

$$\int \frac{3x^2 - 7x + 12}{(x-2)(x^2 - 2x + 5)}\,dx.$$

[4 marks]

A2 Differentiate the following with respect to x;

(a) $y = (\cos x)^{\sin x}$;

(b) $y = \dfrac{1}{(3x^2 - 2x + 1)^5}$.

[3 +3marks]

A3 Solve the equation $z^3 + (1-i)z^2 + (3-i)z - 3i = 0$.

[5 marks]

A4 If $f(x) = \dfrac{3x + |x|}{7x - 5|x|}$ evaluate (where possible);

(a) $Lim_{x \to 0^+} f(x)$ (b) $Lim_{x \to 0^-} f(x)$ (c) $Lim_{x \to 0} f(x)$.

[3+3+3marks]

A5 Find the equation of the plane through the point (-1,4,-3) and perpendicular to the line

$x-3=2t$

$y+3=t$ **[5marks]**

$z-4=-3t$

A6 Apply integration by parts to find;

$$I=\int e^{-2x}\cos 3x dx.$$

[5marks]

A7 Find the derivative of $f(x)=\sin x$ from first principles.

[6marks]

Section B: Answer THREE questions in this section. [60]

B8 (a) (i) Find the exact value of the modulus, r, and the argument,θ of $(1+i)^5$.

(ii) Hence or otherwise express $\frac{(1+i)^5}{(1-i)^7}$ in the form $x+iy$.

[3+4marks]

(b) Apply de Moivre's theorem to evaluate the integral

$$\int e^{3x}\sin 3x dx.$$ **[7marks]**

(c) Suppose $z=Cos\theta+iSin\theta$ show that;

$(z+z^{-1})^4=z^4+4z^2+6+4z^{-2}+z^{-4}$ and deduce that $\cos^4\theta=\frac{1}{8}(\cos 4\theta+4\cos 2\theta+3)$.

[6marks]

B9 (a) The following forces act on a particle at P whose position vector is **3i + j – 2k**

F1= 3i+2j+4k **F2=2i-3j+5k.**

(i) Find the magnitude of the resultant force of the forces acting on the particle

(ii) Find the angle between **F1** and **F2.**

(iii) When **F2** is applied to the particle alone, the object moves from P to a point with position vector **2i+3k**, find the work done. **[9marks]**

(b) (i) Show that the points A(3,2,-1), B(-4,3,2) and C(2,1,-2) are not concurrent.

(ii) Find the equation of the plane containing the three points.

(iii) Find the perpendicular distance of the plane from the origin.

(iv) Find also the area of the triangle formed by the points of the vectors.

[3+3+3+2 marks]

B10 (a) Given that $t = \tan\frac{x}{2}$, show that $\cos x = \frac{1-t^2}{1+t^2}$;

Hence or otherwise evaluate $\int \frac{dx}{2+2\cos x - \sin x}$.

[5+5marks]

(b) Find the two points of intersection of the curves $y^2 = 4x$ and $x^2 = 4y$.

(c) Sketch the two curves.

(d) Find the area enclosed between the two curves.

(e) Find the volume generated when this area is rotated through 360^0 about the y-axies.

[2+2+3+3 marks]

B11 (a) Given that $y^2 + x^2 = 2y\sqrt{1+x^2}$, Use implicit differentiation to find $\frac{dy}{dx}$.

[6marks]

(b) By using the Maclaurin series of e^x and *cos x* or otherwise find the first 3 non zero terms of the MacLaurin series of the function $y = e^{cosx}$.

[7marks]

(c) Sketch the curve $y = \frac{x^2}{(x-1)x-2)}$.

[7marks]

END OF QUESTION PAPER