NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA1116: ENGINEERING MATHEMATICS 1A

July 2005 Supplementary Examination

TIME: 3 HOURS

Attempt ALL questions from section A and ANY THREE questions from section B

Section A: Answer all questions in this section. [40 marks]

A1 Differentiate the following with respect to x,

(a) $y = e^{\ln x}$.

(b) $y = \sqrt[3]{1 + \sin 2x}$.

[2 + 3 marks]

A2 Solve the simultaneous equations

$$\begin{aligned} iz + 2w &= 2 \\ z - (1 + i)w &= 4 \end{aligned}$$

giving z and w in the form a + ib, where a and b are real.

[5 marks]

A3 Evaluate the following limits where possible

(a) $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$, (b) $\lim_{x \to \infty} x^2 e^{-x}$, (c) $\lim_{y \to 0} (1 - y)^{\frac{1}{y}}$.

[3 + 3 + 4 marks]

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A4 Points **A, B, C, D** have position vectors, relative to the origin O, given by

OA = **i** +2**j** − **k**   **OB** = -**i** + 2**j** + c**k**   **OC** = 2**i** +**j** +4**k**
OD = **i** + **j** + **k**

where c is a constant. It is given that **OA** and **OB** are perpendicular.

(i) Find the value of c.

(ii) Show that **OA** is normal to the plane OBC.

[2 + 3 marks]

A5 Use the method of integration by parts to find

$$\int x^2 e^x dx .$$

[5 marks]

A6 Find the derivative of $f(x) = \dfrac{x}{x-1}$ from first principles.

[5 marks]

A7 Show that $\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$, $|x| < 1$.

[5 marks]

Section B: Answer ANY THREE questions from this section [60 marks]

B8 (a) Evaluate the following integrals

(i) $\int x^{-1}\sqrt[3]{1+\ln x}\,dx$.

(ii) $\int_0^{\frac{1}{4}} \left(1-4x^2\right)^{-\frac{1}{2}} \text{dx}$.

[3 + 4 marks]

(b) Show that $\int_0 x\tan^{-1}x\,dx = \dfrac{\pi-2}{4}$.

[7 marks]

(c) Find the volume generated by rotating the curve $y = \sqrt{x}e^{-x}$ through one Revolution about the x-axis between x = 0 and x = 1.

[6 marks]

B9 (a) If $x + y = y^x$, find $\dfrac{dy}{dx}$ in terms of x and y.

[6 marks]

(b) By using repeated differentiation, find the Maclaurin expansion, up to the term in x^2 for $\dfrac{\sin 3x}{1+x^2}$.

[7 marks]

(c) Sketch the curve $y = \dfrac{x^2}{(x-2)(x+1)}$.

[7 marks]

B10 (a) The plane **p** has equation $3x + 2y - z + 1 = 0$ and the line m has equation $\mathbf{r} = (0,10,7) + t(1,3,2)$. The line m intersects **p** at the point A. Find the co-ordinates of A.

[4 marks]

(b) Find the equation of the plane containing the line $x = 4 + 3t$, $y = -t$, $z = 1+5t$ and perpendicular to the plane $x + y + z = 7$.

[7 marks]

(c) Find in parametric form, the line of intersection of the two planes

$$2x - 3y + 4z = 1$$
$$x - y - z = 5$$

Find also the acute angle between these two planes.

[9 marks]

B11 (a) Use DeMoivre's theorem to express $\sin^3\theta$ in terms of $\sin\theta$.

[4 marks]

(b) Use DeMoivre's theorem to show that

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1.$$

[4 marks]

(c) (i) The roots of the equation $z^2 + 2z + 4 = 0$ are denoted by α and β. Find α and β in the form $re^{i\theta}$, giving the exact value of r and θ.

[6 marks]

(ii) Find the exact value of $\alpha\beta^* + \beta\alpha^*$.

[4 marks]

(iii) Using DeMoivre's theorem, or otherwise, show that $\alpha^3 = \beta^3$.

[2 marks]

END OF QUESTION PAPER