NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1116

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA1116: ENGINEERING MATHEMATICS 1A

DECEMBER 2005

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Sections B.

SECTION A: Answer ALL questions in this section [40].

A1. Differentiate the following with respect to x,

(a) $y = (\cos x)^{\sec x}$. [3]

(b) $y = \dfrac{4}{(x^2 + x - 1)^3}$. [3]

A2. Solve $(z+1)^7 + (z-1)^7 = 0$. [5]

A3. Evaluate the following limits

(a) $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{1 + \cos 2x}{(\frac{\pi}{2} - x)^2}$. [5]

(b) $\lim\limits_{x \to \infty} (\sqrt{x+4} - \sqrt{x})\sqrt{x}$. [4]

(c) $\lim\limits_{n \to \infty} \dfrac{3n^2 + \pi}{3n^2 - 2}$. [3]

A4. Find the eqution of the line passing through $(3, 2, -4)$ parallel to the line of intersection of the two planes $x + 3y - 2z = 8$, $x - 3y + z = 0$. [7]

A5. Find the derivative of $\sec x$ from first principles. [5]

A6. Prove that $\text{cosech}^{-1}x = \ln(\frac{1}{x} + \frac{\sqrt{1+x^2}}{x})$, $x \neq 0$. [5]

SECTION B: Answer THREE questions in this section [60].

B7. (a) If $y = yx^{\frac{1}{x}} + y^x$, find $\frac{dy}{dx}$ in terms of x and y. [5]

(b) Sketch the curve

$$y = \frac{x^2}{(x-4)(x+1)}.$$

[6]

(c) Find the Maclaurin expansion, up to term in x^2 for $e^x \sin^{-1}x$. [4]

(d) Find the length of the curve C: $x = t^2, y = t^3, 0 \leq t \leq \sqrt{5}$. [5]

B8. (a) Suppose $z = \cos\theta + i\sin\theta$. Show that

(i) $z^n + z^{-n} = 2\cos n\theta$. [2]

(ii) $z^n - z^{-n} = 2i\sin n\theta$. [2]

(b) Use DeMoivre's theorem to express $\cos^6\theta$ in multiple angles. [4]

(c) Apply DeMoivre's theorem to evaluate the integral

$$\int e^{7x}\sin 3x dx.$$

[7]

(d) Given that $z = 2 + 3i$ is a solution of $z^4 - 3z^3 + 4z^2 + 33z - 65 = 0$. Find all the other roots of the equation. [5]

B9. (a) If $I_n = \int_0^{\frac{\pi}{2}} \cos^n x dx$ show that $I_n = \left(\frac{n-1}{n}\right) I_{n-2}$. [4]

Hence evaluate $\int_0^{\frac{\pi}{2}} \cos^8 x dx$. [4]

(b) Evaluate the following integrals

(i) $\int \frac{3x-5}{2x^2+2x+41}dx.$ [4]

(ii) $\int \frac{dx}{(x+1)\sqrt{x^2+4}}.$ [4]

(iii) $\int \frac{1}{x(x^2+1)}dx.$ [4]

B10. (a) Find the equation of the plane that contains $(2,3,1)$ and the normal vector to the plane given by $4\mathbf{i}+3\mathbf{j}-2\mathbf{k}$. [2]

(b) Find the distance from the point $(2,3,-1)$ to the line

$$\frac{x+1}{1}=\frac{y+2}{1}=\frac{z-2}{-1}.$$

[4]

(c) Find the equation of the plane that contains the line

$$\frac{x-1}{1}=\frac{y-4}{-2}=\frac{z-5}{-1}$$

and is parallel to the line

$$\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}.$$

[7]

(d) Find the point of intersection and the angle of intersection of the line

$$\frac{x-1}{3}=\frac{y+1}{1}=\frac{z-2}{-2}$$

and the plane $2x-3y+z=6$. [7]

END OF QUESTION PAPER