NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA1201

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

CALCULUS OF SEVERAL VARIABLES

APRIL/MAY 2002

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY FOUR** questions from Section B.

SECTION A

A1. If $f(x,y) = x^2 - 2xy + 3y^2$, find (a) $f(-2,3)$; (b) $f(\frac{1}{x}, \frac{2}{y})$; (c) $\frac{f(x,y+k)-f(x,y)}{k}$. [1,2,2]

A2. Sketch and name the surface in 3 dimensional space represented by each of the following: (a) $2x + 4y + 3z = 12$ (b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ [3,3]

A3. Find the limits of the following quotients:

(a) $\lim_{(x,y)\to(1,1):x\neq y} \frac{x^2-2xy+y^2}{x-y}$

(b) $\lim_{(x,y)\to(2,4)} \frac{y+4}{x^2y-xy+4x^2-4x} : x \neq 0, x \neq 1, y \neq -4$

[2,2]

A4. By introducing polar coordinates, show that f is continuous at (0,0) if

$$f(x,y) = \begin{cases} 0 & (x,y) = (0,0) \\ \frac{x^2+y^2}{\ln(x^2+y^2)} & (x,y) \neq (0,0) \end{cases}$$

[3]

A5. The functions u and v of the independent variables x and y are defined implicitly by the system of equations $2u + v = x, u - 3v = y$. Find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}$ [2,2,2]

A6. Evaluate $\int\int_R (x^2+y^2)dxdy$ where R is the region bounded by $y = x^2, x = 1$ and $x = 2$ and the x-axis. [4]

SECTION B

B7. (a) Give the $\epsilon - \delta$ definition of a limit, and hence prove that $\lim_{(x,y)\to(1,2)} xy - 3x + 4 = 3$ [5]

(b) Give the $\epsilon - \delta$ definition of continuity at a point, and hence prove that $f(x, y) = x^2 + 2y$ is continuous at (1,2). [5]

(c) Evaluate (i) $\lim_{(x,y)\to(1,1)} \frac{x^2+y^2}{x-y}$ (ii) $\lim_{(x,y)\to(3,2)} \frac{x^2-y^2}{x^2+2xy+y^2}$ [2,2]

(d) Investigate the continuity of each of the following functions at the indicated points: (i) $x^2 + y^2; (x_0, y_0)$ (ii) $\frac{x}{5x+5y}; (0,0)$ [2,2]

B8. (a) If $F = f(x, y, z)$, find dF. [2]

(b) If $z = x^3 - xy + 3y^2$, compute (i) Δz (ii) dz if $x = 5, y = 4, \Delta x = -0.2, \Delta y = 0.1$ [3,3]

(c) Compute $((3.8)^2 + 2(2.1)^2)^{1/5}$ approximately using differentials. [5]

(d) Show that the expression $(2xy^2 + 3y\cos 3x)dx + (2x^2y + \sin 3x)dy$ is an exact differential of a function. Hence find the function. [5]

B9. (a) Find the equations for (i) tangent plane, (ii) normal line to the surface $x^2yz + 3y^2 = 2xz - 8z$ at $(1, 2, -1)$. [4,2]

(b) Find the equations for (i) tangent line, (ii) normal plane to the curve $3x^2y+y^2z = -2, 2xz - x^2y = 3$ at the point $(1, -1, 1)$. [4,3]

(c) (i) Find the directional derivative of $U = 2x^3y - 3y^2z$ at $\mathbf{P}(1, 2, -1)$ in a direction toward $\mathbf{Q}(3, -1, 5)$. [4]

(ii) What is the magnitude of the maximum directional derivative? [2]

B10. (a) State and verify Green's theorem in the plane for

$$\oint_C (2xy - x^2)dx + (x + y^2)dy$$

where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$ [10]

(b) (i) Prove that $\mathbf{F} = (2xz^3 + 6y)\mathbf{i} + (6x - 2yz)\mathbf{j} + (3x^2z^2 - y^2)\mathbf{k}$ is a conservative force field. [2]

(ii) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is any path from (1,-1,1) to (2,1,-1). [4]

(iii) Give a physical interpretation of the results. [2]

B11. (a) State Stoke's theorem. [4]

(b) Verify Stokes theorem for $\mathbf{A} = 3y\mathbf{i} - xz\mathbf{j} + yz^2\mathbf{k}$, where S is the surface of the paraboloid $2z = x^2 + y^2$ bounded by $z = 2$ and C is its boundary. [14]

B12. (a) Find the maximum values and minimum values of $x^2 + y^2 + z^2$ subject to the constraint $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$ and $z = x + y$. [15]

(b) Give a geometric interpretaion of the above result. [3]

END OF QUESTION PAPER