NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA1201 CALCULUS OF SEVERAL VARIABLES

December 2002

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B

SECTION A

A1. a) Find the domain of definition of each of the following function. Represent the domain on the xy-plane.

$$f(x,y) = \frac{7-xy}{x^2+y^2-4}.$$

[4 marks]

b) If $f(x,y) = 2x^3 - xy - y^2$, find

i) $f(1,-3)$

ii) $f\left(\frac{1}{a},\ 1-b\right)$

[3 marks]

A2. a) Compute the double integral $\iint_D (x+y)$ over the domain D bounded by the curves $y = x$ and $y = x^2$.

[6 marks]

b) Evaluate the triple integral $\iiint z\,dV$ over the domain bounded below by the paraboloid $z = x^2 + y^2$ and above by the sphere $x^2 + y^2 + z^2 = 6$.

[6 marks]

A3. Find the point of conditional extremum of the function $z = x + 2y$ if $x^2 + y^2 = 5$.

[10 marks]

A4. a) If $z = \ln(x^2 + xy + y^2)$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial y}{\partial x} = 2.$

[4 marks]

b) Show that the function $w = f(u,v)$, where $u = x + at$, $v = y + bt$, satisfies the equation:

$$\frac{\partial w}{\partial t} = a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y}.$$

[2 marks]

A5. Find the line integral $\int_L ydx - xdy$, where L is the arch of the cycloid $x = 2(t - \sin t)$, $y = 2(1 - \cos t)$ joining its points $O(0,0)$ and $A(4\pi,0)$.

[8 marks]

A6. a) Find the directional derivative of $u = x^2 + 3xyz + z^3$ at $P(2,3,1)$ in the direction toward $Q(3,4,2)$.

b) What is the equation of the
i) tangent plane
ii) normal line

to the surface with equation $z = x^2 - y^2$ at the point $M(2,1,3)$

[4 marks]

c) Use differentials to approximate $(1.02)^{3.01}$

[5 marks]

SECTION B

B7. i) State Green's Theorem.
ii) Prove Green's Thorem.

[14 marks]

B8. Determine the coordinates of the point A such that the sum of the squares of its distances from three given points O(0,0), P(1,0) and Q(0,1) has the least possible value. Find also the point within the triangle with vertices O, P and Q for which the sum of the squares of its distances to the vertices has the greatest value.

[14 marks]

B9. a) Define the limit of a function $u = f(x_1, ...x_n)$ of n independent variables.

b) Investigate the existence or non-existence of the following limits:

i) $$\lim_{(x,y)\to(0,0)} \left(\frac{x-2y}{x^2+y^2} \right)$$

ii) $$\lim_{(x,y)\to(0,0)} \left(\frac{x^2y^2}{x^2+y^2} \right)$$

iii) Consider the function

$$f(x,y) = \frac{x^2y}{x^2+y^2}.$$

Is the function continuous on the entire xy – plane? If there is any point of discontinuity, how should the function be defined on that point in order to make it continuous?

[14 marks]

B10. a) Define what it means to say a function f is differentiable. The height of a cone is $h = 30cm$, and the base radius is $r = 10cm$. By making use of differentials find how the volume of the cone will change if its height is increased by $3mm$ and its volume reduced by $1mm$.

b) Evaluate the integral:

$$\iint_S xdydz + dxdz + xz^2dxdy$$

were S is the outer side of the part of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

[14 marks]

END OF QUESTION PAPER.