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NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

JULY 2001 SUPPLEMENTARY EXAMINATION

SMA1203 THEORETICAL MECHANICS

July 2001
3 Hours (100 marks) 3 pages

Answer ALL questions in SECTION A and ANY THREE in SECTION B.

SECTION A: Answer all questions [40 marks]

1. Given that the unit base vectors $\underline{e}_r$ and $\underline{e}_\theta$ in polar coordinates (r, θ) are (cos θ, sin θ) and (- sin θ, cos θ) respectively, deduce the components of the acceleration in polar coordinates.

[6 marks]

2. A particle moves on the positive x-axis under a force F per unit mass. F is defined by

$$F(x) = \begin{cases} -\lambda x/a, & x \le a \\ -\lambda a^2/x^2, & x \ge a \end{cases}$$

where λ and a are constants.

If the particle is projected from the origin $(x = 0)$ with a velocity $V\left(= \sqrt{3a\lambda}\right)$, show that its speed at $x(> a)$ is $\sqrt{\left(2\lambda a^2 / x\right)}$.

[6 marks]

3. A particle of mass m is projected vertically upwards from the origin O with a speed U. The particle moves under gravity in a medium which produces a resistance proportional to the velocity. Calculate the time taken for the particle to reach its maximum height.

[5 marks]

4. Calculate the work done by the force

$$\underline{F} = (y + z, x + z, x + y).$$

in moving a particle from the origin O to point A (1,1,1) along the straight line OA.

[5 marks]

5. A system consists of three particles of masses 1, 2 and 3 units respectively. Their position vectors at time t relative to a fixed orthogonal coordinate system are (t,2, 2t), (t^2, 2t,5) and (5, 3t, t^3) respectively. Calculate (a) the centre of mass of the system, (b) the velocity of the centre of mass and (c) the kinetic energy of the system.

[6 marks]

6. State the number of degrees of freedom and a set of generalized coordinates which

specify the motion of the following systems :
(a) a particle moving along a space curve, (b) two particles moving freely on a plane, (c) a rod with one end fixed at a point and is free to move in a vertical plane

[6 marks]

7. Define Hamiltonian H. Show that Hamilton's equations

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

are equivalent to Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

where L is the Lagrangian.

[6 marks]

Section B

Answer ANY three questions This section has 60 marks.

8. A particle of mass m is projected from a point O with a speed u at an angle α to the horizontal. The particle moves under constant gravity g in a medium which produces a resistive force k$\underline{v}$ per unit mass, where k is a constant and $\underline{v}$ is the velocity. Choose a rectangular Cartesian coordinate system Oxy with the y-axis pointing vertically upwards. Find (a) the equation of the trajectory (b) the time taken for the particle to return to the level of projection (y = 0) and (c) the equation of the trajectory in the limit as k $\to$ 0.

[20 marks]

9. Show that the moment of inertia of a square plate OABC about its axis OA is (1/3) m a ^{2}, where a is the length of OA.
The plate OABC has its corner O freely pivoted to a fixed point of a smooth horizontal plane. The plate is initially vertical and is then allowed to fall from rest under gravity with its lower edge sliding on the plane. Write down the equations describing the motion and solve them.

[5+15 marks]

10. Derive Euler's equations which can be written, in the usual notation, as

$$I_1\dot{w}_1 - (I_2 - I_3)w_2 w_3 = M_1$$
$$I_2\dot{w}_2 - (I_3 - I_1)w_1 w_3 = M_2$$
$$I_3\dot{w}_3 - (I_1 - I_2)w_1 w_2 = M_3$$

All the assumptions made should be stated clearly.

An ellipsoid, free to move about its centre, is set in rotation at time *t*=0 with an angular velocity $\underline{w}$. If $(n, 0, 3n)$ are the components of $\underline{w}$ along the principal axes at the centre, and $6I, 3I$ and I are the moment of inertia about these axes. Find the magnitude of $\underline{w}$ as $t \to \infty$.

[7+13 marks]

11. Derive the energy equation from Lagrange's equations in the case of a conservative system with fixed constraints.
Two uniform rods having the same length $2a$ and masses 2m and 3m are smoothly hinged at one end of each and denote this point B. A, the free end of the less massive rod is smoothly hinged to a fixed point, so that the rods are free to swing in a vertical plane under gravity. Denote C the free end of the other rod. If θ and ϕ are the angles that AB and BC make with the vertical respectively, show that the Lagrangian L can be written as

$$L = ma^2[(22/3)\dot{\theta}^2 + 2\dot{\phi}^2 + 6\dot{\theta}\dot{\phi}\cos(\phi - \theta)]$$
$$+ 2mga\cos\theta + 3mga(2\cos\theta + \cos\phi)$$

Assume that $\theta, \phi, \dot{\theta}, \dot{\phi}$ are small , deduce from Lagrange's equations that θ and ϕ satisfy the equations

$$\frac{44}{3}\ddot{\theta} + 6\ddot{\phi} + \frac{8g}{a}\theta = 0$$
$$6\ddot{\theta} + 4\ddot{\phi} + \frac{3g}{a}\phi = 0$$

Obtain the general solution for θ and ϕ.

[5+15 marks]

END