NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

MAY 2002 EXAMINATION

SMA1203 THEORETICAL MECHANICS

May 2002
3 Hours (100 marks)

Answer ALL questions in SECTION A and ANY THREE in SECTION B.

SECTION A: Answer all questions [40 marks]

1. The times taken by a boat to travel a distance s upstream and downstream are t_1 and t_2 respectively. Show that the speed of the boat relative to the water is $s(t_1 + t_2)/2t_1t_2$.

[6 marks]

2. A particle moves on the positive x-axis under a force F per unit mass. F is defined by

$$F(x) = \begin{cases} -\lambda x/a, & x \le a \\ -\lambda a^2/x^2, & x \ge a \end{cases}$$

where λ and a are constants.

If the particle is projected from the origin $(x = 0)$ with a velocity $V(= \sqrt{3a\lambda})$, determine its speed at x. Consider the two cases x < a and x > a..

[6 marks]

3. A particle of mass m is projected vertically upwards from the origin O with a speed U. The particle moves under gravity in a medium which produces a resistance proportional to the velocity. Calculate the time taken for the particle to reach its maximum height.

[5 marks]

4. Show that the force

$$\underline{F} = (y + z, x + z, x + y)$$

is conservative and find the potential V associated with $\underline{F}$.

[5 marks]

5. A system consists of three particles of masses 1, 2 and 3 units respectively. Their position vectors at time t relative to a fixed origin are $\underline{r_1}, \underline{r_2}$ and $\underline{r_3}$ respectively. Suppose that the force acting on each particle consists of an external force and an interparticle force. Write down the equation of motion for each particle and hence deduce that the rate of change of the total linear momentum of the system is independent of the interparticle forces.

[6 marks]

6. State the number of degrees of freedom and a set of generalized coordinates which specify the motion of the following systems :
(a) a particle moving along a space curve, (b) two particles moving freely on a plane, (c) a rod with one end fixed at a point and is free to move in a vertical plane

[6 marks]

7. Define (a) a virtual displacement, (b) a Lagrangian L and (c) a Hamiltonian H.

[6 marks]

Section B

Answer ANY three questions This section has 60 marks.

8. Show that the unit base vectors in polar coordinate system (r, θ) are (cos θ, sin θ) and (- sin θ, cos θ). Deduce the velocity and acceleration components in (r,θ) coordinate system.
A particle of unit mass is projected from a point distant a from the origin with velocity V in a direction perpendicular to the radius vector. Assume that the particle moves under an inverse cube law such that it experiences a force directed towards the origin and of magnitude λ / r^3 where λ is a constant. Determine its path for the case when aV > $\sqrt{\lambda}$.
[Hint: substitute u = 1 / r and solve for u, treating u as a function of θ]

[20 marks]

9. Show that the moment of inertia of a square plate OABC about its axis OA is (1/3) m a^2, where a is the length of OA.

The plate OABC has its corner O freely pivoted to a fixed point of a smooth horizontal plane. The plate is initially vertical and is then allowed to fall from rest under gravity with its lower edge sliding on the plane. Write down the equations describing the motion and solve them.

[5+15 marks]

10. A set of axes Oxyz, with a fixed origin O, is rotating with an angular velocity $\underline{w}$ about an inertial frame. Show that the equation of motion of a particle of mass m under a force $\underline{F}$, when referred to the rotating frame, can be written as

$$m\frac{\partial^2 \underline{r}}{\partial t^2} = \underline{F} - m\left[\frac{\partial \underline{w}}{\partial t} x \underline{r} + 2\underline{w} x \frac{\partial \underline{r}}{\partial t} + \underline{w} x (\underline{w} x \underline{r})\right].$$

A particle of unit mass moves under gravity on a smooth plane inclined at an angle α to the horizontal and rotates at a constant angular speed w about a fixed vertical axis which intersects the plane at O. Choose a set of axes Oxy, fixed in the plane, such that Ox is along the line of greatest slope downwards. If terms of order w^2 can be neglected, show that if the particle starts from rest at O, its deviation from Ox at time t is $(1/6)wgt^3 \sin 2\alpha$.

[8 + 12 marks]

11. Derive the energy equation from Lagrange's equations in the case of a conservative system with fixed constraints.

Two uniform rods having the same length $2a$ and masses 2m and 3m are smoothly hinged at one end of each and denote this point B. A, the free end of the less massive rod is smoothly hinged to a fixed point, so that the rods are free to swing in a vertical plane under gravity. Denote C the free end of the other rod. If θ and ϕ are the angles that AB and BC make with the vertical respectively, show that the Lagrangian L can be written as

$$L = ma^2[(22/3)\dot{\theta}^2 + 2\dot{\phi}^2 + 6\dot{\theta}\dot{\phi}\cos(\phi-\theta)]$$
$$+ 2mga\cos\theta + 3mga(2\cos\theta + \cos\phi)$$

Assume that $\theta, \phi, \dot{\theta}, \dot{\phi}$ are small, deduce from Lagrange's equations that θ and ϕ satisfy the equations

$$\frac{44}{3}\ddot{\theta} + 6\ddot{\phi} + \frac{8g}{a}\theta = 0$$
$$6\ddot{\theta} + 4\ddot{\phi} + \frac{3g}{a}\phi = 0$$

Obtain the general solution for θ and ϕ.

[5+15 marks]

END