NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA1204

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

ORDINARY DIFFERENTIAL EQUATIONS

JUNE 2004

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. When light radiation enters a medium, its rate of absorption with respect to the depth of penetration t is proportional to the amount of light that is incident on a unit area at that depth. Find the law relating L, the quantity of light, and t, if the incident light is L_0 and the emerging light after passing through a thickness t_1 is L_1. [6]

A2. Solve the initial value problem

$$y' = 6\cos^2 x - y\cot x, \quad y(\pi/4) = 3.$$

[6]

A3. Prove that if $a \in \mathbb{R}$, then $e^{at}\mathcal{L}^{-1}\{f(s)\} = \mathcal{L}^{-1}\{f(s-a)\}$. [6]

A4. The differential equation

$$y'' + (\tan x - 2\cot x)y' = 0$$

has integrals

$$y_1 = \sin x - \frac{1}{3}\sin 3x$$

and

$$y_2 = \sin^3 x.$$

Are these solutions linearly independent? [7]

A5. Use the method of undetermined coefficients to solve the differential equation

$$y'' = 9x^2 + 2x - 1.$$

[7]

A6. Consider the initial value problem

$$y' = x + y, \quad y(0) = 1.$$

By applying Picard's iteration scheme, show that the solution is

$$y(x) = 2e^x - x - 1.$$

[8]

SECTION B: Answer THREE questions in this section [60].

B7. (a) Prove that if

$$M(x, y)dx + N(x, y)dy = 0.$$

where $M(x, y)$ and $N(x, y)$ are continuous with continuous partial derivatives. Then there exists a function u such that

$$\frac{\partial u}{\partial x} = M(x, y) \text{ and } \frac{\partial u}{\partial y} = N(x, y)$$

if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

[10]
[10]

(b) Solve the differential equation

$$(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0.$$

[10]

B8. (a) Evaluate

$$\mathcal{L}^{-1}\left\{\frac{1}{(s^2+k^2)^2}\right\}.$$

[6]

(b) Use the Convolution Theorem to evaluate

$$\mathcal{L}^{-1}\left\{\frac{1}{(s^2+1)(s^2+4)}\right\}$$

[6]

(c) Apply inverse Laplace Transforms to solve the initial value problem

$$y''(t)+\beta^2 y(t)=A\sin\omega t,\quad y(0)=1,\quad y'(0)=0$$

where $\beta, \omega \neq 0$.
Consider the cases where

(i) $\omega \neq \beta$;
(ii) $\omega = \beta$.

[8]

B9. Prove that if $y(x)$ is a continuous function, then it is a solution to the initial value problem $\frac{dy}{dx}=f(x,y), y(x_0)=y_0$ if and only if

$$y(x)=y_0+\int_{x_0}^{x} f(t,y(t))dt.$$

[6]

By setting $y=x^k$, find the general solution of

$$x^2y''-6xy'+10y=0.$$

[6]

(b) Show that the equation of the type

$$\frac{dy}{dx}=f(y/x)$$

can be solved by the variables separable technique if the substitution $y/x=v(x)$ is made.
Use this method to solve the initial value value problem:

$$\frac{dy}{dx}=\frac{y}{x}+\frac{2x^3\cos x^2}{y}, y(\sqrt{\pi})=0.$$

[8]

B10. (a) Use the method of variation of parameters to find the general solution of the equation:

$$y'' - y = e^{-2x} \sin e^{-x}.$$

[8]

(b) Find the general solution of the system

$$\frac{dx}{dt} = -4x + 2y$$
$$\frac{dy}{dt} = -x - y$$

and sketch the resulting phase portrait. Find the solution satisfying

$$x(0) = 4, \quad y(0) = 3.$$

[12]

END OF QUESTION PAPER