NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1203

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

ORDINARY DIFFERENTIAL EQUATIONS SUPPLEMENTARY

AUGUST 2004

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. It is assumed that the earth cannot support a poulation greater than 20 billion persons, and the rate of poulation growth is proportional to the difference between how close the world population is to this limiting value. What is the mathematical expression describing the world population as a function of time? [4]

A2. Find the general solution to

$$y\sec^2 x + (\tan x + 2y)\frac{dy}{dx} = 0.$$

[6]

A3. Find the general solution to the differential equation

$$x\frac{dy}{dx} - y = x^2\cos x.$$

[5]

A4. Prove that if $a \in \mathbb{R}$, then $e^{at}\mathcal{L}\{f(s)\} - \mathcal{L}^{-1}\{f(s-a)\}$. [6]

A5. Solve the following initial value problem

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}, \quad y(0) = -1.$$

[6]

A6. Evaluate

(a) $\mathcal{L}^{-1}\{\frac{s}{(s^2+2s+2)^2}\}$.

(b) $\mathcal{L}\{e^{-3t}\sin 4t\}$

[6,7]

SECTION B: Answer FOUR questions in this section [60].

B7. (a) If the population of the earth was found to be 3.5 billion in 1970 and is i increasing at a rate of 2% per year, when will a population of 50 billion be reached? [5]

(b) Solve the initial value problem

$$y' = 6\cos^2 x - y\cot x, \quad y(\pi/4) = 3.$$

[6]

(c) Find the general solution of

$$y'' + \lambda^2 y = \kappa \cos \omega t, \quad \kappa \neq 0$$

when

(i) $\omega \neq \lambda$

(ii) $\omega = \lambda$

and discuss the boundedness of each of the solutions. [9]

B8. (a) Prove that if $y(x)$ is a continuous function, then it is a solution to the initial value problem $\frac{dy}{dx} = f(x,y), y(x_0) = y_0$ if and only if

$$y(x) = y_0 + \int_{x_0}^{x} f(t, y(t))dt.$$

[5]

(b) Solve the initial value problem

$$y^2 - 2x + (2xy^2 + 2xy - 2x^2)\frac{dy}{dx} = 0, \quad y(1) = 1.$$

[6]

(c) Find the general solution of

$$x^2y'' - 6xy' + 10y = 0$$

by setting $y = x^k$. Hence use the method of variation of parameters to find the general solution of

$$x^2y'' - 6xy' + 10y = 3x^4 + 6x^3.$$

[9]

B9. (a) Prove that if $\mathbf{z}$ is a complex solution of $\frac{d\mathbf{z}}{dt} = A\mathbf{z}$, then the real and imaginary parts of $\mathbf{z}$ are real solutions. [8]

(b) Find the general solution of the system

$$\frac{dx}{dt} = x - y$$
$$\frac{dy}{dt} = -2x \qquad (1)$$

and sketch the resulting phase portrait. Find a solution satisfying

$$x(0) = 2, \quad y(0) = 1.$$

[12]

B10. (a) Show that the general solution of the corresponding homogeneous equation of the first order inhomogeneous linear differential equation

$$y' + p(x)y = q(x)$$

is $y = Ce^{-\int p(x)dx}$.
Considering C as a function of x show, by substituting back into the inhomogeneous equation, that $C(x)$ is the solution of

$$\frac{dC}{dx} = q(x)e^{\int p(x)dx},$$

in which the variables are separable.
Integrate this result and find the general solution of the inhomogeneous differential equation. [12]

(b) Use the above analysis to solve the differential equation

$$y' = \frac{2y}{x} + x^2e^x.$$

[8]

END OF QUESTION PAPER

Question No	Qn1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
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