NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1204

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

ORDINARY DIFFERENTIAL EQUATIONS I SUPPLEMENTARY

JULY 2001

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. Solve the ordinary differential equation

$$(2x^3 - 5y^4)\frac{dy}{dx} = -3x^2y^2 - 1, \quad y(1) = 2.$$

[6]

A2. Solve the following initial value problem by applying the principle of undetermined coefficients:

$$y'' - 3y' + 4y = 5 + 4x + 8e^{3x}, \quad y(0) = 2, \quad y'(0) = 1.$$

[7]

A3. Use the Wronskian to show that the general solution of $y^{(iv)} - 2y''' + y'' = 0$ is

$$y = c_1e^x + c_2xe^x + c_3 + c_4x.$$

[6]

A4. Find the first four terms for the initial value problem

$$\frac{dy}{dx} = x + y^2, \quad y(0) = 0$$

by using the Picard iterative scheme.

[6]

A5. Solve the initial value problem

$$y''' - 2y'' + y = 0, \quad y(0) = y'(0) = 0, \; y''(0) = 1.$$

[6]

A6. Use step function notation to re-write the following functions, and hence find the Laplace transform of each function:

(a) $f(t) = |2t - 1|$

(b) $g(t) = \begin{cases} 2 & \text{if} \quad t < 1 \\ 3t & \text{if} \quad 1 \leq t < 2 \\ 5 & \text{if} \quad 2 \geq 2 \end{cases}$

[4,5]

SECTION B: Answer THREE questions in this section [60].

B7. (a) State the convolution theorem, and hence find

(i) $\mathcal{L}^{-1}\left[\frac{1}{(s^2-5s+6)^2}\right]$

(ii) $\mathcal{L}^{-1}\left[\frac{s}{(s^2+1)^2}\right]$

[2,3,4]

(b) Solve the initial value problem by means of the Laplace transform:

$$(D^4 - 1)x = \begin{cases} 2 & , \quad 0 \leq t < 3 \\ 0 & , \quad t \geq 3 \end{cases} \quad x(0) = x'(0) = x''(0) = 0, \quad x'''(0) = 2.$$

[11]