NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1204

DEPARTMENT OF APPLIED MATHEMATICS

SMA1204 ORDINARY DIFFERENTIAL EQUATIONS

SUPPLEMENTARY EXAMINATION JULY 2005

Time : 3 hours

This paper contains TWO sections. Answer ALL questions in section A and THREE questions from section B.

SECTION A: Answer ALL questions in this section [40].

A1. A body at an unknown temperature is placed in a room which is held at a constant temperature of 30^oF. If after 10 minutes the temperature of the body is 0^oF and after 20 minutes the temperature of the body is 15^oF, find the unknown initial temperature. [6]

A2. Solve the differential equation

$$(x^2 - 3y^2)dx + 2xydy = 0.$$

[4]

A3. Find the particular solution of the following differential equations

$$y'' + 4y' + 8y = \sin x, \quad y(0) = 1, \quad y'(0) = 0.$$

[8]

A4. Use the method of variation of parameters to solve the differential equation

$$y''' - 3y'' + 2y' = \frac{e^x}{1+e^{-x}}.$$

[8]

A5. (a) Given that the differential equation

$$(kx^3y^3 - 2xy)dx + (3x^4y^2 - x^2)dy = 0$$

is exact, find the value of k. [2]

(b) Hence find the general solution of the differential equations. [4]

A6. (a) Show that the substitution $w = y^{1-n}$, $n \neq 1, 0$ reduces the equation

$$\frac{dy}{dx} + p(x)y = q(x)y^n$$

to a linear equation of the form

$$\frac{dw}{dx} + (1-n)p(x)w = (1-n)q(x).$$

[4]

(b) Hence solve

$$\frac{dy}{dx} + \frac{1}{x}y = xy^3.$$

[4]

SECTION B: Answer TWO questions in this section [60].

B7. (a) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 9 & 1 \end{pmatrix}.$$

[5]

Hence or otherwise solve the system of differential equations

$$\begin{aligned} \frac{dx}{dt} &= x + y \\ \frac{dy}{dt} &= 9x + y. \end{aligned}$$

[2]

Draw the phase portraits for the system. [3]

(b) The basic equation governing the amount of current I (in amperes) in a simple RL circuit consisting of a resistance R (in ohms), an inductor L (in heries) and an electromotive force (emf) E (in volts) is

$$\frac{dI}{dt} + \frac{R}{L} I = \frac{E}{L}.$$

An RL circuit has an emf given by $3 \sin 2t$, a resistance of 10 ohms, an inductance of 0.5 henry and an initial current of 6 amperes. Find the current at any time. [10]

B8. (a) Show that the general solution of the corresponding homogeneous equation of the first order inhomogeneous linear differential equation

$$y' + p(x)y = q(x)$$

is $y = Ce^{-\int p(x)dx}$. [3]

(b) Considering C as a function of x show, by substituting back into the inhomogeneous linear differential equation, that $C(x)$ is the solution of

$$\frac{dC}{dx} = q(x)e^{\int p(x)dx},$$

in which the variables are separable. [4]

(c) Integrate this result and find the general solution of the inhomogeneous differential equation. [4]

(d) Use the analysis to solve the differential equation

$$y' = y \tan x + \cos x.$$

[9]

B9. (a) Find the general solution of

$$y''' - 2y'' - 5y' + 6y = (e^{2x} + 3)^2.$$

[10]

(b) Solve the system

$$\begin{aligned} \frac{dx}{dt} - x + \frac{dy}{dt} &= 2t + 1 \\ 2\frac{dx}{dt} + x + 2\frac{dy}{dt} &= t. \end{aligned}$$

[10]

B10. (a) Solve the differential equation

$$yy'' + (y')^2 = y^2.$$

[5]

(b) For the differential equation

$$(1 + 4x^2)y'' - 8y = 0,$$

(i) find the singular points, [2]

(ii) find a recurrence formula for the power series solution around $x = 0$ for the differential equations, [8]

(iii) hence find the general solution near $x = 0$. [5]

END OF QUESTION PAPER