SUPPLEMENTARY EXAMINATION

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1205

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

STATISTICS I

MAY/JUNE 2001

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY FOUR** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. For any two events **A** and **B** prove that

$$P[\mathbf{A} \cap \mathbf{B}] = P[\mathbf{A}] + P[\mathbf{B}] - P[\mathbf{A} \cup \mathbf{B}]$$

[4]

A2. Consider two events **A** and **B**. Given that $P[\mathbf{A}] = 0.3$ and $P[\mathbf{A} \cup \mathbf{B}] = 0.7$, find $P[\mathbf{B}]$ if

(a) **A** and **B** are mutually exclusive, [2]

(b) **A** and **B** are independent, [2]

(c) $P[\mathbf{B}|\mathbf{A}] = 0.5$ [2]

A3. Suppose that ten coins are such that if the i^{th} coin is tossed, then a head appears with probability $i/10$, $i = 1, 2, \ldots, 10$ When one coin is selected and tossed, it turns up heads. What is the probability that it was the second coin? [4]

A4. Suppose that the lifetimes of bulbs produced in a certain factory are distributed according to the following probability density function:

$$f(x) = \begin{cases} xe^{-cx} & for \quad c > 0, x > 0 \\ 0 & otherwise \end{cases}$$

(a) Determine the constant c, [3]

(b) Determine the distribution function of X and sketch its corresponding graph, [5]

(c) Determine the following probabilities:
(i) $P[X < 100]$, (ii) $P[X = 100]$, (iii) $P[75 < X < 125]$ [2,2,2]

A5. Suppose X and Y have a joint discrete distribution for which the joint probability function is given by

$$f(x,y) = \begin{cases} 1/30(x+y) & for \quad x = 0,1,2; y = 0,1,2,3 \\ 0 & otherwise \end{cases}$$

(a) Determine the marginal probability functions of X and Y. [2,2]

(b) Are the variables X and Y independent? [3]

A6. Suppose that the life (X hours) of a certain electronic component has the following probability density function:

$$f(x) = \begin{cases} ce^{-3x} & for \quad 0 < x < \infty, c > 0 \\ 0 & otherwise \end{cases}$$

(a) Determine the value of c. [2]

(b) In a random sample of n such electronic components, what is the probability that at most $n-1$ components will last at least an hour? [3]

SECTION B: Answer FOUR questions in this section [60].

B7. Suppose that the joint probability density function of X and Y is as follows:

$$f(x,y) = \begin{cases} c(x^2+y) & for \quad 0 \le y \le x^2 \le 1, \\ 0 & otherwise. \end{cases}$$

(a) Determine c [5]

(b) Find the correlation between X and Y. [10]

B8. Suppose that X and Y have a continuous joint distribution for which the joint probability function is as follows:

$$f(x,y) = \begin{cases} cx^2y & 0 \le x^2 \le y \le 1 \\ 0 & otherwise \end{cases}$$

(a) Determine the value of c, [2]

(b) Find $P[X \ge Y]$, [3]

(c) Find the marginal probability function of X, and hence calculate $P[X \le 1/2]$ [3,2]

(d) Find the marginal probability function of Y, and hence calculate $P[Y \ge 1/2]$ [3,2]

B9. (a) Suppose that the lifetimes, X, of light bulbs produced in a certain factory are distributed according to the following probability density function:

$$f(x) = \begin{cases} xe^{-x} & for \quad x > 0 \\ 0 & otherwise \end{cases}$$

(i) Find the moment generating functon, the mean and the variance of X. [5]

(ii) Find the moment generating function, the mean and the variance of $Y = 3X + 4$ [5]

(b) Suppose that a random variable X has a discrete distribution for which the probability function is

$$f(x) = \begin{cases} \binom{15}{x} 0.5^{15} & for \quad x = 0, 1, 2, \ldots, 15, \\ 0 & otherwise \end{cases}$$

Find the moment generating function, the mean and the variance of X. [5]

B10. Suppose that X and Y have a discrete joint distribution for which the joint probability function is as follows:

$$f(x,y) = \begin{cases} c(2x+y) & for \quad x = 0,1,2 \& y = 0,1,2,3 \\ 0 & otherwise \end{cases}$$

(a) Determine c and $P[X \leq 1 \& Y \geq 2]$. [3]

(b) Find the marginal probability functions of X and Y. Are X and Y independent? [3]

(c) Find the conditional probability function of X given that $Y = 0$. [3]

(d) Determine $P[X \leq 1]$ and $P[X \leq 1 | Y = 0]$ [3,3]

B11. Suppose that X_1, X_2 and X_3 have a discrete joint distribution for which the joint probability function is as follows:

$$f(x_1,x_2,x_3) = \begin{cases} c(x_1+x_2+x_3) & for \quad x_1 = 0,1,2 \& x_2 = 0,1,2 \& x_3 = 0,1,2, \\ 0 & otherwise. \end{cases}$$

(a) Determine c and $P[X_1 \leq 1 \& X_2 \leq 1 \& X_3 \geq 2]$. [3]

(b) Find the marginal probability functions of X_1, X_2 and X_3. Are the random variables independent? [5]

(c) Find the conditional probability function of X_1, X_2 given that $X_3 = 0$. [3]

(d) Determine $P[X_1 \leq 1 \& X_2 \leq 1]$ and $P[X_1 \leq 1 \& X_2 \leq 1 | X_3 = 0]$. Comment on the above answers. [4]