NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA1205 - STATISTICS I

May 2002

Time : 3 Hours

Answer ALL questions in Section A and any FIVE in Section B.

SECTION A: Answer ALL questions in this Section.

1. The probability that a doctor correctly diagnoses a particular illness is 0.7. If the doctor makes an incorrect diagnosis, the probability that the patient sues the doctor is 0.9. What is the probability that the doctor makes an incorrect decision and the patient sues?

[5 marks]

2. The proportion of defective items in a large batch of items is p. Suppose that three items are selected at random from the batch. What is the probability that at least two items are defective?

[5 marks]

3. A large industrial firm uses three local hotels to provide overnight accomodation for its clients. From experience it is known that $x\%$ of the clients are assigned rooms at Holiday Inn, $y\%$ at the Sun Hotel, and $z\%$ at Churchill Arms $(x + y + z = 100)$. If the plumbing is faulty in $p\%$ of the rooms at Holiday Inn, $q\%$ of the rooms at the Sun Hotel, and in $r\%$ of the rooms at Churchill Arms, what is the probability that a client with a room having faulty plumbing was assigned accomodation at the Sun Hotel?

[5 marks]

4. Suppose that the probability function of a certain random variable X has the following form:

$$f(x) = \begin{cases} cx & \text{for } x = 1, 2, 3, 4, 5, \\ 0 & \text{otherwise.} \end{cases}$$

Determine the value of c and hence calculate $P[X > 2]$.

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[5 marks]

5. Suppose that in a certain electrical system the voltage X is a random variable for which the distribution function is as follows:

$$F(x) = \begin{cases} 0 & \text{for } x < 0, \\ \frac{x}{1+x} & \text{for } x \geq 0. \end{cases}$$

Calculate the median of the distribution and $P[100 < X < 200]$.

[5 marks]

SECTION B: Answer FIVE questions in this Section.

6. Suppose that $X_1, X_2, ..., X_n$ form a random sample from a normal distribution with mean μ and variance σ^2. The moment generating function of this normal distribution is given by:

$$\Psi(t) = exp(\mu t + \frac{1}{2}\sigma^2 t^2), \quad -\infty < t < \infty.$$

Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

(a) Show that $\bar{X}$ has a normal distribution with mean μ and variance $\frac{\sigma^2}{n}$.

[10 marks]

(b) Determine an upper bound for $P[|\bar{X} - \mu| \geq 3\frac{\sigma}{\sqrt{n}}]$.

[5 marks]

7. Suppose that the proportion of defective items in a large lot is p, and suppose that a random sample of n items is selected from the lot. Let X denote the number of defective items in the sample, and Y denote the number of nondefective items.

(a) Describe the distributions of X and Y completely. **You should also specify $E[X]$, $Var[X]$, $[Y]$, $Var[Y]$ in your descriptions of the two distributions.** Determine $E[X - Y]$.

[10 marks]

(b) Find a simple expression for $P[Y \geq 2]$ in terms of n and p.

[5 marks]

8. Suppose that the random variables X and Y have a continuous joint distribution for which the joint probability density function is defined as follows:

$$f(x, y) = \begin{cases} c(x^2 + y) & \text{for } 0 \leq y \leq 1 - x^2, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a positive constant.

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(a) Determine the value of c.

[5 marks]

(b) Find the marginal probability density function for X and hence evaluate $P[-0.5 < X < 0.5]$.

[10 marks]

9. Suppose that the random variables X and Y have a discrete joint distribution for which the joint probability function is defined as follows:

$$f(x,y) = \begin{cases} c|x+y| & \text{for } x = -2,-1,0,1,2 \text{ and } y = -2,-1,0,1,2, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a positive constant.

(a) Determine the value of c.

[4 marks]

(b) Find the covariance and the correlation between X and Y.

[11 marks]

10. The moment generating function of a Poisson distribution with mean λ is

$$\Psi(t) = e^{\lambda(e^t-1)} \text{ for } -\infty < t < \infty.$$

(a) Prove that if $X_1, X_2, ..., X_n$ form a random sample from a Poisson distribution with mean λ then the sum $Z = \sum_{i=1}^{n} X_i$ has a Poisson distribution with mean $n\lambda$.

[10 marks]

(b) Show that $P[n\lambda - 2\sqrt{n\lambda} < Z < n\lambda + 2\sqrt{n\lambda}] \geq \frac{3}{4}$.

[5 marks]

11. Suppose that the length of life (X) of an electronic component, measured in hours, has a continuous distribution for which the probability density function is given by

$$f(x) = \begin{cases} 0.001e^{-0.001x} & \text{for } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Determine the mean ($E[X]$), the median and variance ($Var[X]$) of X.

[10 marks]

(b) Determine the probability that the component will last at least 100 hours.

[5 marks]

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END