NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

July 2001 EXAMINATION

SMA1209 COMPUTER PACKAGES IN MATHEMATICS

3 Hours

This paper contains THREE sections. Answer ALL the questions in each section.

SECTION A : Answer ALL questions from this section. Total marks : 40

Use **DERIVE** to answer each of the following questions and write the answer in the answer book provided. At the end of your **DERIVE** session, save your worksheet under the name a:D - - - -.MTH, where - - - - are the last four digits of your registration number.

1. Evaluate
$$\int_0^{\pi/4} \tan(x + \frac{\pi}{4})\, dx.$$
[2 Marks]

2. Find the value of $\frac{d^2y}{dx^2}$ at $x = 1$, given that $y = \frac{e^{-3x} + 1}{\sin \pi x + \log 3x - 1}$.
[2 Marks]

3. Determine
$$\lim_{x \to 1^+} (x^2 - 1) \log (x - 1)^6$$
[2 Marks]

4. Find, to 3 decimal places, the overall minimum value of $\sin x + (x + 7)^2$.
[4 Marks]

SECTION B : Answer ALL questions from this section. Total Marks : 35

Use **MINITAB** to answer each of the following questions and write the answer in the answer book provided. At the **beginning** of your **MINITAB** session, open the OUTFILE 'a:M - - - -.LIS', where - - - - are the last four digits of your registration number.

9. Enter the following data into 2 columns in MINITAB

x	1	2	4	4	5	6	8	9
y	2	4	5	7	8	9	11	13

(a) Find the mean and standard deviation of xy^2.

[5 Marks]

(b) Find the equation of the regression line of y on x.

[5 Marks]

10. The length of steel rods produced by Issels follows a normal distribution with a mean of 1.026 metres and a standard deviation of 0.015 metres. By generating 1000 random numbers, estimate

(a) the proportion of rods with a length in excess of 1.05 metres,

[5 Marks]

(b) the mean length of rods classfied as satisfactory by quality control, that is those rods with a length of between 1 and 1.05 metres.

[5 Marks]

11. Use the SET command to put the numbers 0:1000 into a column.
Hence find the mean difference and the root mean square difference between $\sin(x^2)$ and $(\sin x)^2$ for x taking values from 0 to 1 in steps of 0.001.

[10 Marks]

12. A high jumper runs directly towards the bar and takes off a horizontal distance y before it. The height of the lowest part of his body above the ground follows the equation $h = Ax(B - x)$, where x is the distance along the ground from the take off point and A and B follow normal distributions with means 1.8 and 1.5 and standard deviations 0.2 and 0.3 respectively.

(a) Estimate the number of times out of 1000 jumps that the jumper will clear the bar if it is set at a height of 1.8 metres and y is 0.8 metres.

[5 Marks]

5. Find, to 3 decimal places, the real roots of

$$x^4 - 3x^3 - x^2 + 1 = 0.$$

[4 Marks]

6. Find the first four non-zero terms in the Taylor Series expansion of xe^{-x^2} about $x = 0$.

[3 Marks]

7. Given the matrix $A = \begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 1 \\ -1 & 2 & 3 \end{pmatrix}$

(a) find A^{-1},

[2 Marks]

(b) find the eigenvalues and a set of eigenvectors for A.

[4 Marks]

8. Solve each of the following differential equations.

(a)

$$\frac{dy}{dx} + y = x^3 e^{-x}.$$

[4 Marks]

(b)

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = e^{-t},$$

given that $y(0) = 0$ and that $\frac{dy}{dt}(0) = 1$.

[4 Marks]

SECTION C : Answer ALL questions from this section. Total Marks : 25

Use **EXCEL** to answer each of the following questions and write the answer in the answer book provided. At the end of your **EXCEL** session, save your worksheet under the name a:E - - - -.XLS, where - - - - are the last four digits of your registration number.

13. The following data represents monthly income and expenditure(in dollars) for a company for the first six months of the year 2000.

Month	*Income*	*Expenditure*
Jan	2345	1965
Feb	2543	2209
Mar	2628	1880
Apr	2812	2887
May	2990	2756
Jun	3215	2734

(a) For the next six months, income is expected to rise by 10% each month and expenditure is expected to rise by 12% each month. Hence estimate the total income and total expenditure for the year 2000.

[5 Marks]

(b) Tax will be taken as a percentage of income. Create a column showing the profit after tax. Hence find the mean monthly profit after a tax of 10% and a tax of 15%.

[5 Marks]

(c) Use the Data Analysis Tool to find the median and the skewness of the monthly income.

[5 Marks]

14. Solutions are required to the equation

$$x \sin 2x = e^{-x}(x+1)^3$$

between $x = 0$ and $x = 10$. By plotting graphs of $y = x \sin 2x$ and $y = e^{-x}(x+1)^3$, find these solutions to an accuracy of 0.1. Label the graph and the axes appropriately.

[10 Marks]

END OF EXAMINATION PAPER