NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1211

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

MATHEMATICS FOR SCIENCE II

MAY 2002

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. Find the following limit

$$\lim_{(x,y)\to(0,0)} \frac{e^y \sin x}{x}.$$

[4]

A2. Find the vector equation of the line that passes through a point A with position vector $\mathbf{a} = -2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and is normal to both the vectors $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{c} = -\mathbf{i} + \mathbf{j} - \mathbf{k}$. [6]

A3. Suppose $w = r^2 + sv + t^3$ and $r = x^2 + y^2 + z^2, s = xyz, v = xe^y$ and $t = yz^2$. Use the chain rule to find $\frac{\partial w}{\partial z}$. [5]

A4. Solve the differential equation by applying the principle of undetermined coefficients:

$$y'' - 3y' + 4y = 5 + 4x + 8e^{3x}.$$

[5]

A5.

Change the following Cartesian integral into an equivalent polar integral and evaluate:

$$\int_{-1}^{\ln 2} \int_{0}^{\sqrt{(\ln 2)^2 - y^2}} e^{\sqrt{x^2+y^2}}\, dx dy.$$

[5]

A6. Solve the following initial value problem

$$\frac{dy}{dx} = \frac{1+x^2}{1+y^2}, \quad y(0) = -1.$$

[5]

A7. Find the equation of the tangent to the surface $z^2 = xyz + 3$ at the point (-1,2,-3). [5]

A8.

Solve Kepler's equation

$$M = E - e \sin E$$

by 4 iterations of the Newton-Raphson method for the eccentric anomaly E, given the mean anomaly $M = 0.8$ and eccentricity $e = 0.2$. Use $E_0 = M$ as the initial estimate. [5]

SECTION B: Answer THREE questions in this section [60].

B9. (a) Solve the following initial value problem

$$x\frac{dy}{dx} - y = x^2 \cos x.$$

[6]

(b) A copper ball is heated to a temperature of 100^oC. Then at time $t = 0$ it is immersed in water that is mantained at a temperature of 30^oC. At the end of 3 minutes the temperature of the ball is reduced to 70^oC. Formulate a differential equation given that the rate of change of temperature $T(t)$ is proportional to the difference. Find the time at which the temperature of the ball is reduced to 31^oC. [7]

(c) Use the power series method to solve the following differential equation:

$$y'' - xy = 0.$$

[7]

B10. (a) A simple electrical circuit consists of a resistor R and an electromotive force V. At a certain instant $V = 80$ volts and is increasing at a rate of 5 volts/min. while $R = 40$ ohms and is decreasing at a rate of 2 ohms/min. Use Ohms law, $I = V/R$ and a chain rule to find the rate at which the current I (in amperes) is changing. [5]

(b) Find all the stationary points and classify each as maximum, minimum or saddle point for the function $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$. [7]

(c) By making use of the transformation $u = x + y$ and $v = y - 2x$, evaluate

$$\int_0^1 \int_0^{1-x} \sqrt{x+y}(y - 2x)^2 dy dx.$$

[8]

B11. (a) Is the line $x = 1 - 2t, y = 2 + 5t, z = -3t$ parallel to the plane $2x + y - z = 8$? Give reasons for your answer. [4]

(b) Find the equation of the plane through $(2, 3, 1), (4, 2, 6)$ and $(5, 1, 7)$ and also find where this plane cuts the coordinate axis. [6]

(c) Find the angle between the two planes $3x + 2y - z = 5$ and $x + 4y - z + 1 = 0$. Also find the equations of the planes through the origin

(i) perpendicular to the line of intersection of the given planes;

(ii) through the line of intersection of the given planes. [10]

B12. (a) Consider the initial value problem

$$y' = 1 + xy^2, \quad y(0) = 1.$$

Use the Euler-Cauchy method to compute the approximation to $y(0.2)$ taking $h = 0.1$. [6]

(b) Use Simpson's rule to estimate the integral

$$I = \int_0^{\pi/4} \sqrt{1 - \sin x} dx$$

to 4 decimal places using 6 strips. [6]

(c) Show that the Newton-Raphson method is linearly convergent for a double root. [8]

END OF QUESTION PAPER