

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 1211: MATHEMATICS FOR SCIENCE II

Supplementary Exam

TIME: 3 HOURS

*** LIBRARY USE ONLY ***

Candidates should attempt ALL questions from section A and ANY THREE questions from section B

Section A: Answer all questions in this section. [40]

- A1 Find the equation of the plane through the point $(-1, 4, -3)$ and perpendicular to the line
- $$\begin{aligned}x - 3 &= 2t \\ y + 3 &= t \\ z - 4 &= -3t\end{aligned}$$

[5 marks]

- A2 Given that $z = \ln(x^2 + y^2)$. Verify that the given function satisfies Laplace's equation
- $$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

[6 marks]

- A3 Find the critical point(s) of the function $f(x, y) = 2xy - 5x^2 - 2y^2 + 4x + 4y - 4$ and determine its nature

[6 marks]

- A4 Sketch the region of integration represented by the following integral and evaluate the integral

$$\int_0^4 \int_y^{2y} (8x + e^y) dx dy$$

[6marks]

- A5 Solve the following differential equation
 $y'' + 2y' + 2y = -10xe^x + 5\sin x$

[6marks]

- A6 Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^6 + y^2}$

[5marks]

- A7 Find the volume of the prism whose base is the triangle in the xy -plane bounded by the x -axis and the lines $y = x$ and $x = 1$ and whose top lies in the plane $f(x,y) = 3 - x - y$

[6marks]

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Section B: Answer THREE questions in this section. [60]

- B8 (a) Find in parametric form, the line of intersection of the two planes
 $x - 3y + 2z = 14$
 $-x + y + z = 10$

Find also the acute angle between these two planes

[7marks]

(b) Find the point of intersection of the plane $2x - 3y + 2z = -7$ and the line

$$x = 1 + 2t$$

$$y = 2 - t$$

$$z = -3t$$

[6marks]

(c) Find the equation of the plane containing the line $x = 3t, y = 1+t, z = 2t$ and parallel to the line of intersection of the planes $2x - y + z = 0$ and $y + z + 1 = 0$

[7marks]

B9 (a) Evaluate the integral $\iint_R (2x^2 - xy - y^2) dx dy$ where R is the region bounded by the lines $y = -2x + 4, y = -2x + 7, y = x - 2$ and $y = x + 1$ by using the transformation $u = x - y, v = 2x + y$

[7marks]

(b) Change the Cartesian integral $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy dx$ into an equivalent polar integral and evaluate it

[7marks]

(c) Sketch the region of integration and evaluate the following integral

$$\int_0^1 \int_0^{1-x} (x^2 + y^2) dy dx \quad \int_0^1 \int_0^{1-x} (x^2 + y^2) dx dy$$

[6marks]

B10 The population y of Bulawayo city satisfies the logistic law

$$\frac{dy}{dt} = \frac{1}{100}y \quad \text{where time } t \text{ is measured in years.}$$

(a) Given that the population of Bulawayo was 100 000 in year 2 000, determine the population y as a function of time. [6marks]

(b) What will be the population in year 2020 [5marks]

(c) In what year does the year 2000 population double [5marks]

(d) Assuming the differential equation applies for all $t > 2000$, how large will the population ultimately be [4marks]

B11 (a) Use the power series method to solve the differential equation
 $y'' - xy = 0$

[7marks]

(b) Use the method of undetermined coefficients to solve
 $y'' - y' - 12y = (x+1)e^{2x}$

[7marks]

c) Show that the equation
 $t^2 y'' + (2a+1)ty' + a^2 y = 0$

has solutions $y_1 = t^{-a}$ and $y_2 = t^{-a} \ln t$. Hence find the general solution to
 $t^2 y'' + 5ty' + 4y = 0$

[6marks]

END OF QUESTION PAPER