NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 1211: MATHEMATICS FOR SCIENCE II

December 2002

TIME: 3 HOURS

Candidates should attempt ALL questions from section A and ANY THREE questions from section B

Section A: Answer all questions in this section. [40]

A1 Find the parametric equations for the line of intersection of the planes
$2x - 3y + z = 1$
$x - y - z = 5$

[5 marks]

A2 In an ideal gas PV= nRT, where n is the number of moles of the gas, R is a constant and variables P,V and T denote the pressure , volume and absolute temperature respectively. Show that

$$\frac{\partial P}{\partial V} \times \frac{\partial V}{\partial T} \times \frac{\partial T}{\partial P} = -1$$

[6marks]

A3 Find the solution to the boundary value problem
$y'' - 4y' + 13y = 0$ Subject to $y(0) = -1, y'(0) = 2$

[6 marks]

A4 Evaluate the following limits where possible

(a) $\lim_{(x,y)\to(1,1)} \frac{x^3-y^3}{x^2-y^2}$

(b) $\lim_{(x,y)\to(0,0)} \frac{\sin xy}{xy}$

[3+3marks]

A5 Sketch the region of integration represented by the following integral and evaluate the integral

$$\int_1^2 \int_y^{5-y} e^{x+3y}\,dx\,dy$$

[6marks]

A6 Express $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ in terms of s and t given that
$x = 3s + 5t, y = s - t$ and $f = f(x, y)$

[6marks]

A7 (a) Given that $z = e^{x^2-y^2}\cos 2xy$. Verify that the given function satisfies Laplace's equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

[5marks]

Section B: Answer THREE questions in this section. [60]

B8 (a) Find the equation of the plane, whose points are equidistant from the two points (2,-1,1) and (3,1,5)

[7marks]

(b) Find the equation of the plane that passes through the points (2,3,1), (4,2,6) and (5,1,7). Also find the points where the plane cuts the coordinate axes.

[7marks]

(c) Find the perpendicular distance of the point (2,1,3) from the plane $2x+6y+9z-4=0$ and also find the coordinates of the foot of the perpendicular from this point to the plane.

[6marks]

B9 (a) The molecular concentration $C(x,t)$ of a liquid is given by $C(x,t)=t^{-1/2}e^{-x^2/kt}$. Verify that this function satisfies the diffusion equation

$$\frac{k}{4}\frac{\partial^2 C}{\partial x^2}=\frac{\partial C}{\partial t}$$

[7marks]

(b) Find the equation of the tangent plane and the line normal to the surface

$x^2+y^2-z^2=18$ at the point (3,5,-4)

[7marks]

(c) Find the stationary points and their nature for the following function

$f(x,y)=x^3+y^3-3x^2-3y^2-9x$

[6marks]

B10 (a) Sketch the region of integration and evaluate the following integral

$$\int_0^1 \int_0^{1-x} (x^2 + y^2)\,dy\,dx$$

[6marks]

(b) By applying the transformation $u = \dfrac{2x-y}{2}, v = \dfrac{y}{2}$ evaluate the integral

$$\int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \frac{2x-y}{2}\,dx\,dy$$

[7marks]

(c) Use polar coordinates to evaluate the integral

$$\iint_R e^{-(x^2+y^2)}\,dx\,dy$$

where R is the region in the xy plane bounded by the circle

$x^2 + y^2 = a^2$

[7marks]

B11 (a) Use the power series method to solve the differential equation

$y'' - xy = 0$

[7marks]

(b) Use the method of undetermined coefficients to solve

$y'' - y' - 12y = (x+1)e^{2x}$

[7marks]

c) Show that the equation

$t^2 y'' + (2a+1)ty' + a^2 y = 0$

has solutions $y_1 = t^{-a}$ and $y_2 = t^{-a}\ln t$. Hence find the general solution to

$t^2 y'' + 5ty' + 4y = 0$ **[6marks]**

END OF QUESTION PAPER