

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 1211: MATHEMATICS FOR SCIENCE II

JUNE 2004

TIME: 3 HOURS

Candidates should attempt ALL questions from section A and ANY THREE questions from section B

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**Section A: Answer all questions in this section. [40]**

- A1 Find the equation of the plane through the point  $(-1, 4, -3)$  and perpendicular to the line  
 $x - 3 = 2t$   
 $y + 3 = t$   
 $z - 4 = -3t$

[5 marks]

- A2 Given that  $z = \ln(x^2 + y^2)$ . Verify that the given function satisfies Laplace's equation  
 $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ .

[6 marks]

- A3 Find the critical point of the function  $f(x, y) = 2xy - 5x^2 - 2y^2 + 4x + 4y - 4$  and determine its nature.

[6 marks]

- A4 Evaluate the integral  $\int_0^4 \int_y^{2y} (8x + e^y) dx dy$ .

[6 marks]

- A5 Solve the following differential equation  
 $y'' + 2y' + 2y = -10xe^x + 5 \sin x$ .

[6 marks]

A6 Show that  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y}{x^6 + y^2}$  does not exist.

[5marks]

A7 Evaluate the following

$$\lim_{(x,y) \rightarrow (1,1)} \frac{x^3 - y^3}{x^2 - y^2}$$

[6marks]

**Section B: Answer THREE questions in this section. [60]**

B8 (a) Find in parametric form, the line of intersection of the two planes

$$x - 3y + 2z = 14$$

$$-x + y + z = 10$$

Find also the acute angle between these two planes.

[7marks]

(b) Find the point of intersection of the plane  $2x - 3y + 2z = -7$  and the line

$$x = 1 + 2t$$

$$y = 2 - t$$

$$z = -3t$$

[6marks]

(c) Find the equation of the plane containing the line  $x = 3t, y = 1 + t, z = 2t$  and perpendicular to the line of intersection of the planes  $2x - y + z = 0$  and  $y + z + 1 = 0$ .

[7marks]

- B9 (a) Evaluate the integral  $\int_{-1}^3 \int_{y^2}^{2y^2+3} (x+y) dx dy$ . [10marks]
- (b) Evaluate the following integral  $\int_0^1 \int_0^{1-x} (x^2 + y^2) dy dx$ . [10marks]

B10 The population  $y$  of Bulawayo City satisfies the logistic law

$$\frac{dy}{dt} = \frac{1}{100} y, \text{ where, time } t \text{ is measured in years.}$$

- (a) Given that the population of Bulawayo was 100 000 in year 2 000, determine the population  $y$  as a function of time. [6marks]
- (b) What will be the population in year 2020 [5marks]
- (c) In what year does the year 2000 population double [5marks]
- (d) Assuming the differential equation applies for all  $t > 2000$ , how large will the population ultimately be. [4marks]
- B11 (a) Solve the differential equation  $y'' - 4y' + 13y = 0$  subject to  $y(0) = -1, y'(0) = 2$ . [7marks]
- (b) Use the method of undetermined coefficients to solve  $y'' - y' - 12y = (x+1)e^{2x}$ . [7marks]
- (c) Find the extremum of  $f(x, y) = x^3 + y^3 - 3x^2 - 3y^2 - 9x$ . [6marks]

**END OF QUESTION PAPER**