NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA1211 : MATHEMATICS FOR SCIENCE II

SUPPLEMENTARY EXAMINATION

JULY 2005
3 Hours

Answer ALL questions from this Section A and any THREE questions in Section B

SECTION A : Answer ALL questions from this section [40 marks]

A1. If $f(x,y) = x^3 - 2xy + 3y^2$, find
(i) $f(2,3)$
(ii) $f(1/x, 2/y)$ [4]

A2. Find the angle between $\bar{a} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\bar{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ [4]

A3. If a line l has parametric equations

$$x = 3t + 1, \quad y = -2t + 4, \quad z = t - 3$$

find the equation of the plane that contains l and the point P(5, 2, 0). [5]

A4. Find the stationary points of the surface defined by

$$z = f(x, y) = x^2 - y^2 - 2xy + 6$$

and determine their nature. [5]

A5. By changing the order of integration, evaluate $\int_0^1 \int_y^1 \cos x^2 dxdy$. [5]

A6. Let $u(x, y) = x^2 \sin y$ where $x = s^2 + t^2$ and $y = 2st$. Use the chain rule to find $\frac{\partial u}{\partial s}$ and $\frac{\partial u}{\partial t}$. [4]

A7. Obtain the Euler approximation to the solution of the initial value problem

$$y' = 2y, \quad y(0) = 1$$

on [0 , 0.15] using h = 0.05. Compare with the exact solution. [6]

A8. Solve the following initial value problem

$$(x^2 + 1)\frac{dy}{dx} + 4xy = x, \quad y(2) = 1$$ [7]

SECTION B: Answer any THREE questions [60 marks]

B9. (a) Evaluate $\iint_D xy\,dxdy$ where D is the region in the first quadrant bounded by the lines $y = 1$, $x = 0$ and the curve $y = x^2$.

(b) Evaluate $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} y\,dydx$

(c) Use the change of variables $u = y - x$ and $v = x + y$ to evaluate the integral $\iint_R \cos\left(\frac{y-x}{y+x}\right)dA$ where R is the trapezoidal region with vertices (1,0), (2,0) (0,2) and (0,1).

[5 , 5 , 10]

B10. (a) Find the general solution to the equation

$$\frac{dy}{dx} = \frac{xy - y^2}{x^2}.$$

(b) Solve the differential equation

$$\frac{dy}{dx} - y\tan x = \sin x.$$

(c) Uranium disintegrates at a rate proportional to the amount present at any instant. If M_0, M_1 and M_2 grams of uranium are present at time $t = 0, t = T_1$ and $t = T_2$ respectively, show that the half – life of uranium is $\dfrac{(T_2 - T_1)\ln 2}{\ln\left(M_1/M_2\right)}$.

[4, 6, 10]

B11. (a) Find a, b, c if $(a+b-2)\hat{i}+(c-1)\hat{j}+(a+c)\hat{k}=0$.

(b) Find where the line $\frac{x-3}{2}=\frac{y+1}{4}=\frac{z-5}{3}$ cuts the plane $2x+3y-z=11$.

(c) Find the equation of the plane through (2,3,1), (4,2,6) and (5,1,7) and also find where this plane cuts the co – ordinate axes.

(d) Find parametric equations for the line of intersection of the planes

$$-2x+3y+7z+2=0 \text{ and } x+2y-3z+5=0.$$

[3, 5, 6, 6]

B12. (a) Find the solution to the boundary value problem

$$y''-4y'+13y=0 \text{ subject to } y(0)=-1,\ y'(0)=2.$$

(b) By applying the Runge - Kutta method (with one iteration) to the differential equation

$$y'=1+y^2, \quad and \quad y(0)=0,$$

estimate the value of $y(0.1)$.

(c) Use the improved Euler method to estimate $y(0.1)$, and $y(0.2)$ from the differential equation : $\frac{dy}{dx}=x^2+4,\ \ y(0)=2.$

[6, 7, 7]

B13. (a) The rate of flow of gas in a pipe is given by $v = Cd^{1/2}T^{-5/6}$, where C is constant, d is the pipe diameter and T is the absolute gas temperature. The measurement of d is subject to a maximum percentage error of ±1.6% and that of T to one of ±1.2%. Find approximately the maximum percentage error in the value of v.

(b) Show that

$$(3x^2y+x^2-2y^2)dx+(x^3+y-4xy+6y^2)dy$$

can be written as exact differential of a function $\phi(x,y)$ and find this function.

(c) In an experiment to investigate Newton's law of cooling, a copper ball is heated to a temperature of $100^{\circ}C$. Then, at time $t = 0$, it is placed in water that is maintained at $30^{\circ}C$. At the end of 3 minutes the temperature of the ball has reduced to $70^{\circ}C$.

(i) Formulate an ordinary differential equation given that the rate of change of temperature $T(t)$ at any time, t, is proportional to the difference $T - 30$.

(ii) Find the time at which the temperature of the ball is reduced to $31^{\circ}C$.

[5, 5, 10]

END OF QUESTION PAPER