NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1216

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA 1216: ENGINEERING MATHEMATICS 1B

DECEMBER 2002

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B. Candidates are advised to start each question on a fresh page.

SECTION A: [25 marks]

Answer **ALL** questions from this section.

A1. (a) Calculate $\begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \phi} \end{vmatrix}$

if $x = r\cos\phi,\ y = r\sin\phi$ [3]

(b) The angle between any two surfaces is defined as the angle between their tangent planes at any point P. Calculate the angle between the cylinder $x^2 + y^2 = R^2$ and the sphere $(x - R)^2 + y^2 + z^2 = R^2$ at the point $M\left(\frac{R}{2}, \frac{R\sqrt{3}}{2}, 0\right)$. [5]

A2. (a) Find the Maclaurin series, up to and including third order terms, of $f(x, y) = e^x \sin y$. [5]

(b) Calculate and identify the stationary points of $f(x, y, z) = xyz$ subject to

$$\begin{aligned} x + y + z &= 5 \\ xy + yz + zx &= 8 \end{aligned}$$

[8]

A3. Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{pmatrix}$ [4]

SECTION B: [75 marks]

Answer ANY THREE questions from this section. Each question carries 25 marks.

B4. (a) Find the values of t for which the determinant of the matrix

$$B = \begin{pmatrix} t-1 & 3 & -3 \\ -3 & t+5 & -3 \\ -6 & 6 & t-4 \end{pmatrix}$$ is equal to zero. [10]

(b) Solve the system of equations

$$\begin{aligned} x + 2y - 3z + 3w &= 2 \\ 2x + 5y - 8z + 6w &= 5 \\ 3x + 4y - 5z + 2w &= 4 \end{aligned}$$

[9]

(c) Determine whether or not the vectors

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} \right\}$$ are linearly dependent or not. [6]

B5. (a) Determine whether or not the vectors

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix} \right\}$$ form a basis for $\mathbb{R}^3$. [7]

(b) Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{pmatrix}$$ [5]

(c) Find all the real eigenvalues and eigenvectors of

$$C = \begin{pmatrix} -3 & 1 & -1 \\ -7 & 5 & -1 \\ -6 & 6 & -2 \end{pmatrix}$$ [13]

B6. (a) Solve the following differential equations

(i) $\tan x \dfrac{dy}{dx} = y$ [3]

(ii) $\dfrac{dy}{dx} = -\dfrac{x+y}{x}$ [4]

(iii) $\dfrac{dy}{dx} = y \tan x + \cos x$ [5]

(b) Show that the equation
$(x+y)dx + (x+2y)dy = 0$ is exact. [1]
Hence find its solution. [5]

(c) Solve the equation
$$y'' + 2y' + y = \frac{e^{-x}}{x}$$ [7]

B7. (a) Solve the system $\begin{cases} \frac{dy}{dx} + 2y + 4z = 1 + 4x \\ \frac{dz}{dx} + y - z = \frac{3}{2}x^2 \end{cases}$ [7]

(b) A particle is projected from the ground at an angle α with the horizontal. If its initial velocity is v_0 and air resistance is proportional to its velocity, find the equation of motion of the particle. [10]

(c) Solve the equation
$$(1+y^2)dx = \left(\sqrt{1+y^2}\sin y - xy\right) dy$$ [8]

END OF QUESTION PAPER