NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA 1216: ENGINEERING MATHEMATICS 1B

SUPPLEMENTARY EXAMINATIONS JULY 2003
Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B. Candidates are advised to start each question on a fresh page.

SECTION A [25 marks]

Answer **ALL** questions from this section.

A1. (a) Find the rank of the matrix

$$A = \begin{pmatrix} 21 & -3 & 17 & 13 \\ 46 & 11 & 52 & 14 \\ 33 & 48 & 71 & -23 \end{pmatrix}$$

[2]

(b) Find the inverse of

$$B = \begin{pmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{pmatrix}$$

[3]

A2. (a) What conditions must be placed on a, b and c so that the system of equations

$$\begin{aligned} x + 2y - 3z &= a \\ 2x + 6y - 11z &= b \\ x - 2y + 7z &= c \end{aligned}$$

has

(i) no solution

(ii) a unique solution [2]

(iii) infinite solutions [2]

[2]

(b) The paths followed by the banks of a river (in a certain area) are given by the parabola $y = x^2$ and the line $x - y - 2 = 0$. A company wants to construct the shortest possible bridge over this river. Through which points on the banks of the river should the bridge pass? [6]

A3. (a) The height of a cone is H = 30cm, the base radius is R = 10cm. What is the change in volume of the cone if H increases by 3mm and R reduces by 1mm? [4]

(b) Given that $\vec{u} = \vec{i} + \vec{j} - \vec{k}$; $\vec{v} = 2\vec{i} - 3\vec{j} + \vec{k}$ and $\vec{w} = 8\vec{i} - 7\vec{j} + \vec{k}$, determine whether the vectors $\vec{u}, \vec{v}, \vec{w}$ are linearly dependent or not. [4]

SECTION B [75 marks]

Answer **ANY THREE** questions from this section. Each question carries 25 marks.

B4. (a) Calculate the equations of the tangent plane and normal line to the surface given by the equation $x^2 + y^2 + z^2 = 2Rz$ at the point $(R\cos\alpha; R\sin\alpha; R)$ where R is a constant and α is any angle. [5]

(b) Show that the function $u(x, y) = \ln\left(\frac{1}{r}\right)$ where $r = \sqrt{(x-a)^2 + (y-b)^2}$, satisfies the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. [6]

(c) Consider the matrix

$$C = \begin{pmatrix} t+3 & -1 & 1 \\ 5 & t-3 & 1 \\ 6 & -6 & t+4 \end{pmatrix}.$$

Find the value(s) of t such that C is singular. [8]

(d) Calculate the gradient of $z = \sqrt{x^2 - y^2}$ at the point (5;3). [6]

B5. (a) Find the particular solution for the differential equation

$$\frac{dr}{d\theta} = \frac{\sin\theta + e^{2r}\sin\theta}{3e^r + e^r\cos\theta};$$

given that $r = 0$ when $\theta = \frac{\pi}{2}$ [6]

(c) Solve the system of equations

$$\frac{d^2x}{dt^2} + \frac{dy}{dx} + x = y$$
$$\frac{d^2y}{dt^2} + \frac{dx}{dt} - y = -x$$

[12]

B6. (a) A 10-kilogramme mass is attached to a spring which is thereby stretched 0,7 metres from its natural length. The mass is started in motion from the equilibrium position with an initial velocity of $1m/s$ in the upward direction. Find the resulting motion if the force due to air resistance is $-90\frac{dx}{dt}$ newtons. [9]

(b) Solve the boundary value problem

(i) $\frac{d^2y}{dx^2} + 4y = 0$, $0 < x < \pi$; $y(0) = 0$, $y(\pi) = 4$ [5]

(ii) $\sin^2 y dx + \cos^2 x dy = 0$; $y(\frac{\pi}{4}) = \frac{\pi}{4}$ [4]

(c) A certain compound X is formed by the combination of 2 parts of a chemical U with 3 parts of a chemical W. When certain amounts of U and W are placed together, the rate at which X is produced is constantly proportional to the product of the amounts of U and W that are still present at that instant. Determine the amount of X that is produced in time t if $10kg$ of U and $8kg$ of W are placed together at time $t = 0$ and the amount of X at $t = 1$ is $2kg$. [7]

B7. (a) A body of mass m is projected upward in the air with velocity v_0. Air resistance is proportional to its velocity. [7]

(i) Find the motion of the body.

(ii) Show that the maximum height attained is

$$h_{max} = \frac{mv_0}{k} - \frac{m^2g}{k^2}\ln(1 + \frac{kv_0}{mg})$$

[9]

where k is a constant of proportionality. [3]

(iii) What happens when $k \to 0$?

(b) The population of a certain country is known to increase at the rate proportional to the number of people presently living in that country. If, after two years, the population has doubled and after three years the population is 20000, find the number of people initially living in the country. [6]

... radioactive nuclei decays is proportional to the number of such
... the original number of the

(i) What percentage of the original radioactive nuclei will remain after 4500 years? [4]

(ii) In how many years will only one-tenth of the original number remain? [3]

(b) A tank initially holds 100 litres of a brine solution containing 20g of salt. At time $t = 0$, fresh water is poured into the tank at the rate of 5 litres per minute, while the well stired mixture leaves the tank at the same rate. Find the amount of salt in the tank at any time t. [7]

(c) What is the time required for one Zimbabwe dollar to double when invested at the rate of 20% per annum? [*Assume that interest is compounded continuously*] [5]

(d) According to Newton's law of cooling, the rate at which a body loses heat and thereafter, the change in temperature, is proportional to the difference in temperature between the body and the surrounding medium. Let T be the temperature of the body at any time $t \geq 0$ and let T_0 (considered constant) be the temperature of the surrounding medium.

Show that $T = T_0 + (T_1 - T_0)e^{-kt}$ where T_1 is the value of T at $t = 0$ and k is a constant of proportionality. [6]