NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA1216 ENGINEERING MATHEMATICS 1B

MAY 2004

Time : 3 hours

This paper contains **TWO** sections. Answer **ALL** questions in section A and **THREE** questions from section B.

SECTION A: Answer ALL questions in this section [40].

A1. It is given that $F = 2x^3y - 3y^2z$ and that A and B are points whose coordinates are $(1, 2, -1)$ and $(3, -1, 5)$ respectively.

(a) Find the directional derivative of F at A in the direction AB. [6]

(b) What is the magnitude of the maximum directional derivative? [2]

A2. The total surface area S of a cone of base radius r and perpendicular height h is given by

$$S = \pi r^2 + \pi r\sqrt{(r^2 + h^2)}.$$

If r and h are each increasing at the rate of 0.25 cm/sec, find the rate at which S is increasing at the instant when $r = 3$ cm and $h = 4$ cm. [5]

A3. (a) Find the adjoint of the matrix

$$P = \begin{pmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{pmatrix}.$$

[7]

(b) Find the eigenvalues and corresponding eigenvectors for

$$Q = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 4 & -1 \\ -1 & 2 & 0 \end{pmatrix}.$$

[5]

A4. Determine whether or not the following vectors in R^3 are linearly dependent or linearly independent :

$$(1,-2,1), \quad (5,6,-1), \quad (3,2,1).$$

[5]

A5. (a) A particle of mass m is projected vertically upward with initial speed U in a medium in which the resistance is mkv^2, where v is the particle speed at t.

(i) Show that the equation of motion is

$$\frac{dv}{dt} = -(g + kv^2).$$

[1]

(ii) Hence, show also that the particle speed is given by

$$v = c\tan\left\{\tan^{-1}\left[\frac{U}{c}\right] - \frac{gt}{c}\right\}$$

where $c = \sqrt{\frac{g}{k}}$. [4]

(b) Find the general solution of the equation $y'' + 2y' - 8y = 14e^{3x}$. [5]

SECTION B: Answer THREE questions in this section [60].

B6. (a) It is given that

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 3 & 6 & a & 12 \end{pmatrix}$$

(i) Reduce A to row echelon form. [4]

(ii) Hence, or otherwise, find the determinant of A. [2]

(iii) For what real value of a is the matrix singular? [2]

(iv) You may use the information in (i) and (ii) to solve the following system of linear equations with $a = 10$:

$$\begin{aligned} x+y+z+w &= 3 \\ x+2y+3z+4w &= -3 \\ x+3y+6z+10w &= 3 \\ 3x+6y+10z+12w &= -3 \end{aligned}$$

[6]

(b) Prove that if $A = [a_{ij}]$ is of order $m \times n$ and $B = [b_{ij}]$ is of order $n \times p$, then $(AB)^t = B^t A^t$ [6]

B7. (a) For the system of equations

$$\begin{aligned} x+2y &= z \\ 2x+(3+k)y &= 3z \\ (k-1)x+4y &= 3z \end{aligned}$$

(i) find values of k for the system to have a non-trivial solution? [4]

(ii) Find the general solution for each of these values of k. [5]

(b) Show that the substitution $z = y^{1-n}, n \neq 0, 1$ reduces the equation

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

to a linear equation of the form

$$\frac{dz}{dx} + (1-n)P(x)z = (1-n)Q(x).$$

Hence, solve the equation

$$\frac{dy}{dx} - \frac{y}{x} = xy^3.$$

[5]

(c) Find the tangent plane and normal line to the surface

$$3x^2y^3z - 2xy^2z^2 = 10$$

at the point (-1,2,1). [4;2]

B8. (a) Show that the equation

$$(ax^3y^3 - 2xy)dx + (3x^4y^2 - x^2)dy = 0$$

is exact when $a = 4$. Hence, solve the equation when $a = 4$. [5]

(b) Find the absolute maximum and minimum values of

$$f(x, y) = 3xy - 6x - 3y + 7$$

on the closed triangular region R with vertices (0,0), (3,0) and (0,5) [6]

(c) A builder wishes to design a rectangular house containing V cubic meters of heated space so as to minimize heating costs. One wall of the building is to face south. The annual heating costs are estimated to be \$4 per square metre of floor space, \$3 per square meter for all exterior wall not facing south and \$2 per square meter for exterior wall space facing south. Using Lagrange Multipliers find the dimensions that will produce the most energy-efficient building. [9]

B9. (a) In the investigation of a homicide or accident death it is often important to estimate the time of death. From experimental observations it is known that, to an accuracy satisfactory in many circumstances, the surface temperature of an object changes at a rate proportional to the difference between the temperature of the object and that of the surrounding environment (the ambient temperature). This is known as Newton's law of cooling. If $\theta(t)$ is the temperature of the object at time t, and T is the constant ambient temperature, show that θ satisfies the linear differential equation

$$\frac{d\theta}{dt} = -k(\theta - T),$$

where $k > 0$ is a constant of proportionality.

Suppose that at time $t = 0$ a corpse is discovered, and that its temperature is measured to be θ_o. Assuming that at the time of death the body temperature had the normal value of 37^oC and that the above differential equation is valid, determine the time of death, t_d, given that the temperature of the corpse is 29^oC when discovered and 27^oC two hours later, and that the ambient temperature is 21^oC. [8]

(b) Find the general solution of the differential equation:

$$\frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}.$$

[5]

(c) Find the particular solution of

$$EIy'' = \frac{w}{2}(l - x)^2,$$

where E, I, w and l are constants, subject to the condition $y(0) = 0, \quad y'(0) = 0$. Hence find the value of y when $x = l$. [7]

END OF QUESTION PAPER