NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA1216 : ENGINEERING MATHEMATICS 1B

MAY 2005
3 Hours

Answer ALL questions from this Section A and any THREE questions in Section B

SECTION A : Answer ALL questions from this section [40 marks]

A1. A closed cylinder has dimensions r = 5cm, h = 10cm. Find the approximate increase in the total surface area when r increases by 0.2cm and h decreases by 0.1cm. Give your answer in exact form.

[5 marks]

A2. For the function $f(x, y, z) = xy^2z^3$, find the directional derivative at the point $(1, -2, 1)$ in the direction $\hat{i} - \hat{j} + \hat{k}$.

[6 marks]

A3. Find the equations of the tangent plane and normal line to

$$x^2 - 2y^2 - 3z^2 + xyz = 4$$

at the point $(3, -2, -1)$.

[6 marks]

A4. Determine whether or not the following vectors in R^3 are linearly dependent or linearly independent :

$$(2, -1, 4) \;,\; (3, 6, 2) \;,\; (2, 10, -4).$$

[5 marks]

A5. (a) Find the adjoint of the matrix $T = \begin{pmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{pmatrix}$

(b) Hence find T^{-1}.

[7 + 3 = 10 marks]

A6. Show that the differential equation of the type

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

can be solved by the variables - separable technique if we make the substitution

$\frac{y}{x} = v$, that is, $y = vx$.

Use this method to find the general solution of

$$xy' = x\sec\left(\frac{y}{x}\right) + y.$$

[8 marks]

SECTION B: Answer any THREE questions [60 marks]

B7. (a) Find and classify the stationary points of the function

$$f(x, y) = 2x^3 - 9x^2y + 12xy^2 - 60y.$$

(b) A running track is set out in the form of a rectangle, of length L and width W, with two semi - circular areas, of radius $\frac{1}{2}$W, adjoined at each end of the rectangle. If the perimeter of the whole track is fixed at 400m, determine the values of L and W that maximize the area of the rectangle.

[10 + 10 = 20 marks]

B8. (a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{pmatrix}.$$

(b) For what values of k and m does the system

$$\begin{aligned} 2x - y + 4z &= 1 \\ x + 2y - z &= m \\ 3x - y + kz &= -2 \end{aligned}$$

have

(i) a unique solution (ii) no solution (iii) infinitely many solutions?

In the case that it has infinitely many solutions, find them.

[8 + 12 = 20 marks]

B9. (a) Show that the equation

$(\cos x \sin x - xy^2)dx + (y - x^2 y)dy = 0$ is exact.

Hence find its solution.

(b) Solve the following initial value problem

$$(x^2 + 1)\frac{dy}{dx} + 4xy = x, \quad y(2) = 1.$$

(c) When a switch is closed in a circuit containing a voltage source E, a resistance R and an inductance L, the current i builds up at a rate given by

$$L\frac{di}{dt} + Ri = E.$$

Find i as a function of t. How long will it be, before the current reaches one half of its maximum value if $E = 6$ volts, $R = 100$ ohms and $L = 0.1$ henry?

[6 + 6 + 8 = 20 marks]

B10. (a) Find the general solution of the differential equation

$$y'' + 2y' + 10y = 0.$$

(b) By making the substitution $z = -\frac{1}{2y^2}$ solve the differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = \frac{y^3}{x^3}$$

(c) Solve

$$y'' + 2y' + 2y = -10xe^x + 5\sin x, \quad y(0) = 1 \quad , \quad y'(0) = 0.$$

[4 + 6 + 10 = 20 marks]

END OF QUESTION PAPER