

NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

SMA1216 : ENGINEERING MATHEMATICS 1B

SUPPLEMENTARY EXAM JULY 2005

3 Hours

Answer ALL questions from this Section A and any THREE questions in Section B

SECTION A : Answer ALL questions from this section [40 marks]

A1. (a) If $z = \frac{2x - y}{4x + y}$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

(b) If $z = (x + y)f\left(\frac{x}{y}\right)$, find $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$, giving your answer in the simplest form.

[2 + 3 = 5 marks]

A2. Find the equation of the tangent plane and normal lines to $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$ at the point $(-2, 1, -3)$.

[6 marks]

A3. Find the direction at the point $(2, -1, 5)$ in which the function $f(x, y, z) = x^2 y (z - 4)^3$ has its maximum directional derivative, and find this maximum.

[6 marks]

A4. Determine whether or not the following vectors in \mathbb{R}^3 are linearly dependent or linearly independent :

$(2, -1, 4)$, $(3, 6, 2)$, $(2, 10, -4)$.

[5 marks]

A5. (a) Find the adjoint of the matrix $T = \begin{pmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{pmatrix}$.

(b) Hence find T^{-1} .

[7 + 3 = 10 marks]

A6. Show that the differential equation of the type

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

can be solved by the variables - separable technique if we make the substitution

$$\frac{y}{x} = v, \text{ i.e. } y = vx.$$

Use this method to find the general solution of

$$xy' = x \sec\left(\frac{y}{x}\right) + y.$$

[8 marks]

SECTION B: Answer any THREE questions [60 marks]

B7. (a) Find and classify the local extrema of $f(x, y) = 3x^2y + y^3 - 3y^2 + 2$

(b) A cage for a snake is to be built in the shape of a box with one glass face, the remaining five sides being of wood, and should have a volume of 12m^3 . If the glass costs twice as much per square metre as the wood, use the method of Lagrange multipliers to find the dimensions that will minimize the total cost for materials.

[10 + 10 = 20 marks]

B8. (a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{pmatrix}$$

(b) For what values of k and m does the system

$$2x - y + 4z = 1$$

$$x + 2y - z = m$$

$$3x - y + kz = -2$$

have

(i) a unique solution (ii) no solution (iii) infinitely many solutions?

In the case that it has infinitely many solutions, find them.

[8 + 12 = 20 marks]

B9. (a) Show that the equation

$$(\cos x \sin x - xy^2)dx + (y - x^2y)dy = 0 \text{ is exact.}$$

Hence find its solution.

(b) Solve the following initial value problem

$$(x^2 + 1) \frac{dy}{dx} + 4xy = x, \quad y(2) = 1.$$

(c) When a switch is closed in a circuit containing a voltage source E , a resistance R and an inductance L , the current i builds up at a rate given by

$$L \frac{di}{dt} + Ri = E.$$

Find i as a function of t . How long will it be, before the current reaches one half of its maximum value if $E = 6$ volts, $R = 100$ ohms and $L = 0.1$ henry?

[6 + 6 + 8 = 20 marks]

B10. (a) Find the general solution of the differential equation

$$y'' + 2y' + 10y = 0.$$

(b) By making the substitution $z = -\frac{1}{2y^2}$ solve the differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = \frac{y^3}{x^3}$$

(c) Solve

$$y'' + 2y' + 2y = -10xe^x + 5\sin x, \quad y(0) = 1, \quad y'(0) = 0.$$

[4 + 6 + 10 = 20 marks]

END OF QUESTION PAPER