NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

SMA 1216

FACULTY OF APPLIED SCIENCES

DEPARTMENT OF APPLIED MATHEMATICS

ENGINEERING MATHEMATICS 1B

MAY 2006

Time : 3 hours

Candidates should attempt **ALL** questions from Section A and **ANY THREE** questions from Section B.

SECTION A: Answer ALL questions in this section [40].

A1. The pressure P of an enclosed ideal gas is given by

$$P = k\left(\frac{T}{V}\right)$$

where V is the volume, T is the temperature and k is a constant. Given that the percentage errors in measuring T and V are at most 0.6 % and 0.8 % respectively, find the approximate maximum percentage error in P. [5]

A2. For the function

$$F(x, y, z) = xy^2z^3 + (x-3)^5(y+2)^7(z-1)^{11}$$

find the equations for the tangent plane and normal line at the point $(3, -2, 1)$. [6]

A3. Given the function $f(x, y, z) = x + \dfrac{x-y}{y-z}$ find $f_x + f_y + f_z$ in its simplest form. [6]

A4. Determine whether or not the following vectors in R^3 are linearly dependent or independent:

$$[1, 3, 2]\,, \quad [-3, 11, 4]\,, \quad [-2, 4, 1]\,.$$

[5]

A5. (a) Find the adjoint of the matrix

$$B = \begin{pmatrix} 5 & 2 & 4 \\ 3 & -1 & 2 \\ 1 & 4 & -3 \end{pmatrix}$$

[6]

(b) Hence find B^{-1}. [4]

A6. By making a suitable substitution, show that the differential equation

$$\frac{dy}{dx} = \frac{1 - 3x - 3y}{1 + x + y}$$

can be solved by the variables - separable technique. Hence find the general solution. [8]

SECTION B: Answer THREE questions in this section [60].

B7. (a) Find and classify the stationary points of the function

$$f(x, y) = x^3 + y^3 - 63x - 63y + 12xy$$

(b) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface on the unit sphere $x^2 + y^2 + z^2 = 1$. [12,8]

B8. (a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$T = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

(b) For what values of λ and m does the system

$$\begin{aligned} 2x - y + 4z &= 1 \\ x + 2y - z &= m \\ 6x - 2y + \lambda z &= -4 \end{aligned}$$

have

(i) a unique solution?

(ii) no solution?

(iii) infinitely many solutions?
In the case that it has infinitely many solutions, find them. [8,12]

B9. (a) What conditions on the constants a, b, e and f must be satisfied for the differential equation

$$(ax + bt)\frac{dx}{dt} + ex + ft = 0$$

to be exact, and what is the solution of the equation when they are satisfied?

(b) Solve

$$y' - \frac{4}{(x+2)}y = (x+2)^5$$

subject to the condition $y(0) = 8$.

(c) Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference of temperature between the object and the surroundings. Suppose water at a temperature of 100^0C cools to 80^0C in 10 minutes, in a room maintained at a temperature of 30^0C, find when the temperature will become 40^0C. [6,6,8]

B10. (a) Use the method of variation of parameters to find a general solution to

$$y'' - 2y' + y = \frac{e^t}{t^2+1}$$

. [10]

(b) Use the method of undetermined coefficients to find the general solution of

$$y'' + 4y = \sin x,$$

with $y\,(0) = 1$, $y'\,(0) = 1$. [10]

END OF QUESTION PAPER