NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SMA2103 THEORETICAL MECHANICS

NOVEMBER/DECEMBER 2002 EXAMINATION

TIME: 3 HOURS **This paper has 3 Pages**

SECTION A: Answer **ALL** questions **[40 Marks]**

1. A man can swim at a speed of 40m/minute in still water. Find how long he will swim across a river 150 metres wide and flowing at a speed of 30 m/minute, if he directs his course so as to reach a point on the apposite bank 15 metres below his starting point. **[6 marks]**

2. A particle is constrained to move along the equiangular spiral $r = a e^{b\theta}$ so that the radius vector moves with constant angular velocity w. Determine the velocity and acceleration components. **[6 marks]**

3. Show that the force field $\mathbf{F} = e^{y} \sin y\,\mathbf{i} + e^{x} \cos y\,\mathbf{j} + \mathbf{k}$ is conservative and find the potential V associated with $\mathbf{F}$. **[5 marks]**

4. A particle of mass m is projected vertically upwards from the origin O with speed U. The particle moves under gravity in a medium which produces a resistance proportional to mv^2, where v is the speed of the particle. Calculate its maximum height. **[5 marks]**

5. Evaluate $\int_C \mathbf{F}.d\mathbf{r}$ where $\mathbf{F} = (x - 3y)\,\mathbf{i} + (y - 2x)\mathbf{j}$ and C is a closed curve in the xy-plane, and $x = 2\cos\theta$, $y = 3\sin\theta$ from $\theta = 0$ to $\theta = 2\pi$. Also give a physical interpretation. **[6 marks]**

6. Define (a) a virtual displacement
(b) a Lagrangian L
(c) a Hamiltonian H.

[6 marks]

7. An xyz coordinate system is rotating with respect to an XYZ coordinate system having the same origin and assumed to be fixed in space (ie it is an inertial system). The angular velocity of the xyz system relative to XYZ system is given by
$\mathbf{w} = 2t\mathbf{i} - t^2\mathbf{j} + (2t + 4)\,\mathbf{k}$ where t is the time. The position vector of a particle at time t is observed in the xyz system and is given by $\mathbf{r} = (t^2 + 1)\,\mathbf{i} - 6t\mathbf{j} + 4t^3\mathbf{k}$. Find:

(a) the apparent velocity
(b) the true velocity at time t =1

[6 marks]

SECTION B

Answer **ANY THREE** questions. **[60 marks]**.

8. A rigid body consists of three particles of masses 2, 1, 4 located at (1, -1, 1) , (2, 0, 2) and (-1, 1, 0) respectively. Find:

(a) angular momentum of the body if it is rotated about the origin with angular velocity

$$\mathbf{w} = 3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}.$$

(b) the moments of inertia about x, y and z axes.

(c) the kinetic energy of rotation for the system.

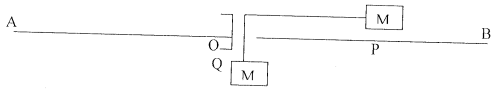
[20 marks]

9. Two ships, s_1 and s_2 are observed at midnight to have the following velocity u_i and position r_i vectors where $i = 1, 2$..

ship s_1: $\mathbf{U}_1 = 4\mathbf{i} + 3\mathbf{j}$; $\mathbf{r}_1 = 4\mathbf{i} - 2\mathbf{j}$
ship s_2: $\mathbf{U}_2 = 6\mathbf{i} + 12\mathbf{j}$; $\mathbf{r}_2 = -20\mathbf{i} - 20\mathbf{j}$

where $\mathbf{i}$, $\mathbf{j}$ are unit vectors in the directions east and north, respectively, and where the speeds are measured in kilometres per hour and distances in kilometres. Determine the time at which the ships would collide if they both continued on their respective courses (i.e. their velocities remained constant) Where would the collision occur?

[20 marks]



10. AB represents a frictionless horizontal plane having a small opening at O. A string of length l which passes through O has at its ends a particle P of mass m and a particle Q of mass m which hangs freely. The particle is given an initial velocity of magnitude V_0 at right angles to string OP when the length OP = a. Let r be the instantaneous distance OP while Q is the angle between OP and some fixed line through O.

(a) Set up the Lagrangian of the system.
(b) Write a differential equation for the motion of P in terms of r.
(c) Find the speed of P at any position.

11. Show that the moments of inertia of a disc about its axis is $\frac{1}{2}Ma^2$

Masses M_1 and M_2 ($M_1 > M_2$) are attached to the ends of a string which passes over a pulley of radius a and mass M, with its axis horizontal and rotating in frictionless bearings. The pulley is sufficiently rough to prevent the string from slipping. Initially the masses hang vertically. Prove that when the masses are released from rest the angular acceleration of the pulley is $\frac{2(M_1 - M_2)g}{(M + 2M_1 + 2M_2)a}$.

The tensions in the strings are T_1 and T_2. Prove that $\frac{T_1}{T_2} = \frac{M_1(M + 4M_2)}{M_2(M + 4M_1)}$

[5 + 15 marks]

END OF QUESTION PAPER.