NATIONAL UNIVERSITY OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF APPLIED MATHEMATICS

SUPPLEMENTARY EXAMINATION JUNE 2003

SMA2103 THEORETICAL MECHANICS

3 hours
June 2003

Answer ALL questions in Section A and any THREE questions in Section B

SECTION A: Answer ALL questions from this section.

1. The times taken by a boat to travel a distance d upstream and downstream are t_1 and t_2 respectively. Show that the speed of the boat relative to the water is $d\,(t_1 + t_2)/\,2\,t_1 t_2$.

[7 marks]

2. Show that the force $\overline{F} = (y + z, x + z, x + y)$ is conservative and find the potential V associated with $\overline{F}$.

[6 marks]

3. A particle of mass M is projected vertically upwards from the origin O with speed U. The particle moves under gravity in a medium which produces a resistance proportional to MV^2 where V is the speed of the particle. Calculate its maximum height.

[7 marks]

4. Prove that in polar coordinates (r,θ), , the velocity and acceleration are given by

$\dot{\underline{r}} = \dot{r}\underline{r} + r\dot{\theta}\hat{\underline{\theta}}$

$\ddot{\underline{r}} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\underline{r}} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\underline{\theta}}$ respectively where $\hat{\underline{r}}$ and $\hat{\underline{\theta}}$ are unit vectors.

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[6 marks]

5. Evaluate $\oint_C \overline{F}.d\overline{r}$ where $\overline{F} = (x-3y)\underline{i} + (y-2x)\underline{j}$

and C is a closed curve in the xy-plane, and $x = 2\cos\theta,\ y = 3\sin\theta$ from $\theta = 0$ to $\theta = 2\pi$. Also give a physical interpretation.

[7 marks]

6. Define (a) a viatnal displacement
   (b) a Lagrangian
   (c) a Hamiltonian H

[7 marks])

SECTION B

ANSWER ANY <u>THREE</u> QUESTIONS (60)

7. Two ships A and B are observed at midnight to have the following velocity v_i and position r_i vectors where $i = 1,2$.

Ship A: $V_1 = 4\underline{i} + 3\underline{j}$; ; $r_1 = 4\underline{i} - 2\underline{j}$

Ship B: $V_2 = 6\underline{i} + 12\underline{j}$; $r_2 = -20\underline{i} - 20\underline{j}$

where $\underline{i}$ and $\underline{j}$ are unit vectors in the directions east and north, respectively, and where the speeds are measured in kilometres per hour and distances in kilometres. Determine the time at which ships would collide if they both confirmed on their respective courses (i.e. their velocities remain constant). Where would the collision occur.

[20 marks]

8. a) Show that the moment of inertia of a square place OABC about its axis OA is $\frac{1}{3}ma^2$, where a is the length of OA.

   b) A light square lamina of side e 2 metres has masses of 1, 2, 3, and 4 kg placed at its corners A, B, C and D respectively. Find the moment of inertia of the system and the radius of gyration
      i. about an axis along AB
      ii. about an axis along BD.

[5+15 marks]

9. A rigid body consists of three particles of masses 2,1,4 located at (1, -1,1) (2, 0. 2) and (-1, 0, 1) respectively.

Find
a) Angular momentum of the body if it is rotated about the origin with angular

velocity $\overline{w} = 3\underline{i} + 2\underline{j} + 4\underline{k}$.

b) the moments of intertia about x, y and z axes.

c) the kinetic energy of rotation for the system.

[20 marks]

10. a) Two particles M_1, M_2 are joined by a light inelastic stain of arc initially close together over a smooth table. A smooth ring of mass M is threaded by the string and hangs over the edge of the table are at right angles to the edge. The mass M_1 moves a distance x towards the edge and the mass M_2 moves a distance y towards the edge. Show that the acceleration of the mass M is $\dfrac{\ddot{x} + \ddot{y}}{2}$ downwards?

Write down equations of motion for three masses and show that the acceleration of the ring is given by $\dfrac{(M_1 + M_2)Mg}{(M_1 + M_2)M + 4M_1M_2}$

[10 marks]

b) A small stone of mass M is thrown vertically upwards with initial speed V. If the air resistance at speed V is mkv where k is a positive constant. Show that the stone returns to its starting point with speed U given by the equation

$$g - kU = (g + kV)\ \exp\left\{\frac{-k}{g}(U + V)\right\}.$$

[10 marks]

END OF QUESTION PAPER.